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Ph.D. Thesis

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Statistical analysis of long hydrological and climatological data series

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I declare that the work presented in this thesis was carried out independently by me, and only with the cited sources, literature and other professional sources.

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In Prague on 29 June 2016 Ondřej Ledvinka

Title: Statistical analysis of long hydrological and climatological data series

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Abstract: Although entitled more generally, the thesis deals primarily with trend analyses applied to the instrumental records of hydrometeorological variables measured over the territory of Czechia, sometimes specializing in particular river basins extending to the neighbouring countries such as Germany and Poland. The hydrological data (namely discharge or spring yield) predominate, to which also the climatological ones such as precipitation, snow cover depth and air temperature can be added since they significantly influence the water cycle in Czechia. Under ideal circumstances, the trend analysis might answer the question whether frequently discussed climate change has its important role in the development of quantitative water resources characteristics. However, due to the record length, starting usually in the 1960s here, it is really hard to conclude if the discovered patterns are a result of deterministic relationships or if they are rather of random origins (e.g. they are a part of climate fluctuation cycle). Trends were identified mainly using the Mann–Kendall test and its modifications whose rationales are studied here in detail. Special focus was on long-term persistence that, besides short-term persistence, may adversely influence the variance of the test statistic. The detection of long-term persistence was carried out via unit root tests that were accompanied by the Hurst exponent estimation. Regarding the precipitation series, there was found no evidence of long-term persistence. However, the hydrological data reveal that the practitioners should be aware of the issues associated with long-term persistence in some areas. The thesis is compiled from several papers published in peer-reviewed journals, showing at its end that the trend analysis may be used as an identification tool also in other fields of hydrology (e.g. return levels estimation). For the purpose of writing the papers, the author mostly used his own `R` scripts that allowed for the introduction of few novel techniques that were applied for the first time in hydrology. Clearly, the topic that is currently under intensive development, cannot be covered exhaustively only in one Ph.D. thesis and the research on it should continue without a doubt, ideally in scientific teams.

Keywords: statistical hydrology, time series analysis, stochastic processes, `R` statistical software, nonstationarity

Název práce: Statistická analýza dlouhých hydrologických a klimatologických řad

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Abstrakt: Přes obecnější název práce je její hlavní součástí analýza trendové složky aplikovaná na instrumentální záznamy hydrometeorologických veličin měřených na celém území Česka. Někdy je ovšem cíleno na povodí přesahující hranice s Německem nebo Polskem. Převažuje analýza hydrologických dat, jako jsou průtoky nebo vydatnosti pramenů, ale byly analyzovány též řady úhrnů srážek, výšky sněhové pokrývky nebo teploty vzduchu, které velmi souvisejí s koloběhem vody odehrávajícím se na území Česka i mimo něj. Za ideálních podmínek může analýza trendu odpovědět na otázku, zda je vývoj kvantitativních charakteristik vodních zdrojů v Česku významně ovlivňován klimatickou změnou. Nicméně, z důvodu délky časových řad, zde většinou začínajících v 60. letech, je obtížné tuto otázku uspokojivě uzavřít s tím, že by bylo možné jistě určit, jde-li o deterministické vztahy či o vliv náhody (např. může jít pouze o část cyklu kolísavého klimatu). Trendy byly detekovány pomocí Mannova–Kendallova testu a jeho modifikací, jejichž podstata zavádění je v práci hojně studována, neboť krátká či dlouhá paměť v analyzovaných časových řadách má za následek, že rozptyl testové statistiky je ovlivněn do takové míry, že test může trend identifikovat nesprávně. Zdali k tomu může docházet i v Česku, bylo zjišťováno pomocí testů na jednotkový kořen, které byly doplněny o odhady Hurstova exponentu. Srážkové úhrny zjevně dlouhou paměť nevykazují, zatímco hydrologické řady v některých oblastech ano, což by se mělo promítnout i do hydrologické praxe. Práce je kompilátem článků, které autor publikoval v recenzovaných časopisech. Články z posední doby studia ukazují, že analýzu trendu lze uplatnit i v jiných oblastech hydrologie, jako je odvozování N-letých hodnot extrémních jevů. Při výpočtech si autor vytvořil několik vlastních skriptů určených pro statistický program `R`, což umožnilo vývoj technik, které byly aplikovány ve světové hydrologii vůbec poprvé. Ukazuje se však, že dané téma zažívá prudký rozvoj a není tedy v moci jediné disertační práce postihnout vše, co se analýzy časových řad v hydrologii týká. Ideální by bylo ve výzkumu pokračovat, přičemž předkládanou práci je nutno považovat za pouhý počátek a základní kámen tohoto výzkumu.

Klíčová slova: statistická hydrologie, analýza časových řad, stochastické procesy, statistický program `R`, nestacionarita

Contents

Preface

"Statistics is a bitch, everyone can rape her."

Jaromír Korčák (1895–1989)
Czech geographer, demographer
and statistician

The above quote characterizes well how not only the student learning statistical methods before their applications may feel. Statistics in general evolved to such an extent that definitely does not allow one to master everything introduced to it. Mainly its applications in different scientific fields stood behind the birth of completely new branches of science such as biostatistics, econometrics, chemometrics, psychometrics, environmetrics and so on. This in turn triggered the development of new methods/techniques exactly tailored to the specific needs in those fields. Sometimes, different terminology is used for the same things, which naturally hinders the self-teaching process. Even if one studies very tied earth sciences such as hydrology and climatology, where statistics plays an important role, very good examples emerge. For instance, *the Nash–Sutcliffe coefficient*, often used in hydrology when assessing the model performance (Nash and Sutcliffe, 1970), is called *the modelling efficiency* in climatology or, more generally, in geophysics (see e.g. v. Buttlar et al., 2014).

Focused very soon just on the applications of statistics in hydrology and climatology while trained in physical geography as a master student at the Faculty of Science, I quickly realized that physical geographers here, despite educated in geographical information systems (GIS) or remote sensing in higher grades, have really a small number of courses devoted to statistics that is, on the contrary, no less important for their future work, especially when coupled with GIS applications or programming skills allowing the development of new analytical tools. Therefore, I joined social geographers and demographers and attended their (geo)statistical courses before finishing my diploma thesis. However, after the graduation I still felt that I needed more in-depth knowledge of statistics to understand the methodological parts of the papers which I read. For my work notably, I needed the time series analysis (TSA) that is a particular tool used by hydrologists when dealing with climate change (CC) or anthropogenic impacts on water resources and with the prediction of their future evolution in several strategic areas.

Exactly this was the reason why I decided to deepen my statistical skills as a Ph.D. candidate. Since I noticed that the Faculty of Science offered a programme well-suited to my needs (i.e. the specialization for scientists without stronger mathematical background), I started to study here and further specialized in data processing and mathematical modelling in science, which might have been promising according to the practical applications performed all over the world. At the same time, I started to work at the Czech Hydrometeorological Institute (CHMI) as an independent hydrologists. Fortunately, the department, where I have been employed for almost eight years, is responsible for maintaining the regime hydrological database containing the majority of digitized hydrological data obtained from the whole territory of Czechia. Also, the CHMI climatologists provided me with access to their database called CLIDATA. It might be more than evident that the combination of such circumstances, specifically bearing in mind the data policy of the CHMI, represented a great opportunity for the initiation of young statistical hydrologist early career. I even did not expect, as I started this way, that this will take so long.

At first, the objective of this thesis was to develop a stochastic model (eventually models) of the Box–Jenkins family capable of predicting daily river discharge fed by different hydrological and climatological data (hereinafter generally hydrometeorological data) in daily time step. Although there are well-observed areas in Czechia for these purposes, several factors prevented me doing so

- Being predominantly self-taught, I must admit that the endless lack of knowledge of these models and differences between several types of stochastic models incorporating besides the predicted variables also the external variables (e.g. groundwater or climate) would certainly lead to the further undesirable prolongation of my study incompatible with success.

- In spite of the fact that I found a huge amount of statistical literature dealing with these models (dynamic linear models, state-space models, ARMAX models, transfer function-noise models, etc.) and also many freely available statistical packages dedicated to these models, I was still not conversant with this issue without any further help from more experienced hydrologists. For example, mixing terminology was really frustrating until I discovered HYNDSIGHT, an excellent Rob J. Hyndman's blog, but it was too late.

- The experts usually surrounding me at my work could not help at all because the CHMI is rather a service than a research institute. Instead, they invited me to cooperate in several grant projects whose focus was on the applications of statistics which had little in common with these models. This made me deal with other statistical issues necessary in the current Czech hydrological service. Primarily, there was a need (and still is) to investigate hydrological drought propagation in Czechia based on the instrumental record.

- The position of Czechia in hydrological science prevented me from gaining the necessary literature from abroad in time. Indeed, many papers, published, for instance, in journals such as Water Resources Research, Journal of Hydrology (mainly papers from the 1980s), or those applying the information embargo (e.g. Hydrological Sciences Journal or Climate Research), could not be gathered until the end of my study.

- Studying CC prior to the applications of these models is fundamental since it may induce nonstationarity in the geosystems such as river basins and may lead to improper setting of the models considering stationarity only (see e.g. McCuen, 2003). However, there is still no dataset in Czechia similar to, for example, the Reference Hydrometric Basin Network (RHBN) used in Canda for investigating CC effects on water resources. Many data here undergo thorough revision and (more or less *ad hoc*) digitization, which of course somewhat hinders the applications of the Box–Jenkins methodology here.

The above list clearly shows that the establishment of the desired stochastic model had to be postponed to the future, for which I really would like to apologize, but it was inevitable. This thesis, therefore, should rather be considered a preliminary work necessary for the inspection if there is a distinct signal of CC and its relation to water resources in Czechia (notably rivers and well-springs). Not all the aspects are studies here. The thesis is rather organized as a collection of papers authored or co-authored by me and published in peer-reviewed journals. The primary focus is on hydrological drought and a trend analysis of various hydrometeorological time series using different nonparametric tests, some of them proposed by me. Anyway, more details can be found in the accompanying chapters before the attachments that are the papers themselves further referred to as A1 to A9 for the sake of brevity.

1. General introduction

1.1 Time series analysis and hydrology

In fact, a lot of data (on either the quantity or quality of water) encountered in hydrology form time series. This is due to the nature of hydrology as a science itself where such data are necessary for modelling the future development of a measured (sometimes ungauged as well) system (e.g. Beven, 2012; Blöschl et al., 2013). Indeed, based on this information then water managers can design various measures often dedicated to the protection of human society and its assets from weather extremes and consequently also from the extremes in hydrological systems as such (e.g. floods and droughts). Studying these extremes is also closely connected with water supply, irrigation, sanitation, wastewater treatment and other sectors where water plays an important role for mankind or other organisms all around the world.

Data collection became a routine after the establishment of hydrological and climatological services in different countries (but somewhere probably much earlier). The need for processing these data naturally led to the use of statistics and especially the TSA in hydrology, which in turn gave the birth to fields today known as statistical hydrology, stochastic hydrology or hydroinformatics. Initially, hydrologists borrowed methods from other sciences such as physics but it may be stated that immediately after the World War II (i.e. in the time of rapid rise of interest in the construction of computers) hydrology became an equal partner of mathematics and physics. It can be documented by the seminal paper from a British hydrologist Harold Edwin Hurst (Hurst, 1951) which inspired famous mathematicians like Benoit Mandelbrot who even contributed to the newly established Water Resources Research journal in the 1960s (see e.g. Mandelbrot and Wallis, 1968, 1969a,b,c,d,e). Since then, the methods developed in hydrology have motivated many others, from climatologists through geophysicists to stock market analysts.

Currently, the specific TSA methods applied in hydrology are developed so quickly that, according to the author's opinion, it is no longer possible to master them all. When dealing with the data analysis, each hydrologist selects his/her favourite techniques ranging from those based on the time domain (addressed mainly here further) and the frequency domain (e.g. harmonic analysis, spectral analysis) to their mixtures (e.g. wavelet analysis; see Sang et al., 2013; Szolgayová et al., 2014) and combining TSA with the Bayesian inference (e.g. Makarava, 2012) and the methods of artificial intelligence (e.g. neural networks and genetic algorithms; see Havlíček et al., 2013; Nacházel et al., 2004). Among the newest methods having a great potential in hydrology belong the nonparametric estimators based on singular spectrum analysis (SSA; e.g. Kondrashov and Ghil, 2006; v. Buttlar et al., 2014) or empirical mode decomposition (EMD; e.g. Sang et al., 2014). From this it is evident that the present thesis could not cover everything and only some of the aspects had to be chosen.

1.2 Thesis objectives

During the past decades and mainly nowadays, the question regarding CC and its effects on water resources has been addressed across the world through the lens of statistics. Thus it seemed quite interesting to look this way at the situation in the territory of Czechia which the author is familiar with as a former geographer. Predominantly the trend analysis is employed when trying to answer whether there are some significant impacts of CC on water resources (including precipitation, snow and evapotranspiration) in different regions. Although several studies were already performed in Czechia or Slovakia, in many cases inappropriate methods were applied, dominated by visual inspections of plots or the linear regression coefficient testing (e.g. Bažatová and Šimková, 2015; Blahušiaková and Matoušková, 2015; Fiala, 2008; Kliment and Matoušková, 2006, 2008, 2009; Majerčáková et al., 1997; Střeštík et al., 2014; Zeleňáková et al., 2012, 2014a,b, 2015a,b). In general, two reasons may be pointed out here (e.g. Hirsch and Slack, 1984; Hirsch et al., 1982; Yue et al., 2002b)

1. The hydrometeorological data do not respect the requirements of the linear regression application (see later in Chapter 3.5.1).
2. The need of independence among data is often violated even for annual data.

Despite the latter condition applies also for common nonparametric techniques, there were developed several modifications that reduce the sensitivity of the original tests. Many of these modifications were in fact developed just in hydrology that focuses more on the nonparametric tests because they usually do not require checking the assumptions before their application and no standardization is needed in most of the cases when the analyst is not forced to quantify the results in physical units. The main focus of the thesis was therefore on these nonparametric methods, the issues connected with the coexistence of a deterministic trend and stochastic persistence (sometimes called trend as well; e.g. Fatichi et al., 2009) in a hydrometeoroloical time series, and the application of modifications of such methods capable of distinguishing between deterministic trends and stochastic (autocorrelation) patterns usually present in time series.

Studying such patterns often requires long time series observed, say of 50 annual entries (e.g. Hosking, 1981). However, this condition is fulfilled only somewhere as regards Czech hydrometeorological data and sometimes shorter time series had to be taken into account when performing nation-wide investigation. Even though there are efforts to get longer series using the so-called proxy data in Czechia (e.g. Elleder, 2015), here, only the instrumental record not exceeding 100 years was considered due to the comparison purposes.

1.3 General mathematical background

To allow better understanding of the subject discussed throughout the thesis and because of the fact that some of the methodology sections in the papers (notably A1 and A2) are described very briefly, it is necessary to formalize somewhat in these preceding chapters. At this point, first, it is necessary to explain what exactly is considered under the term *time series*. A lot of textbooks on hydrology or climatology contain a chapter devoted to TSA which demonstrate its importance (e.g. Eslamian, 2014; Haan, 2002; Maidment, 1993; Shahin et al., 1993; McCuen, 2003; von Storch and Zwiers, 2001; Wilks, 2011). Nevertheless, it is clear that in these chapters the authors mostly presume that their readers are familiar with the basics of TSA and prefer to write about stochastic modelling immediately where the time series were already detrended, deseasonalized (i.e. the deterministic component is suppressed) or centred (i.e. mean equals zero). For instance, the whole books Hipel and McLeod (1994) and Salas et al. (1980) are more or less entirely dedicated to the Box–Jenkins approach (briefly also described in Chapter 3.6).

Probably, recently published book by Machiwal and Jha (2012) starts with the best introduction to TSA in hydrology. Adopting the thoughts of Haan (2002) and Shahin et al. (1993), they define time series as "a sequence of values collected over time on a particular variable" and emphasize the additive structure of whatever time series x_t as follows

$$x_t = Tr_t + Seas_t + \varepsilon_t + \eta_t \tag{1.1}$$

with components Tr_t (i.e. deterministic trend), $Seas_t$ (i.e. seasonality), ε_t (i.e. dependent or correlated stochastic pattern), and η_t (i.e. independent or uncorrelated residual component). It is worth noting that only regularly sampled series are addressed here with the time index, $t = 1, 2, \ldots, T$, determining the place of an element of a series ordered increasingly from its initial value ($t = 1$) to its last value ($t = T$). We will also assume that the seasonal (monthly) series represents $K \cdot 12$ months, where 12 is the largest monthly index from $m = 1, 2, \ldots, 12$ and the index $k = 1, 2, \ldots, K$ represents years. The situation is very similar for daily data where a special index $i = 1, 2, \ldots, I$ will be used (indeed, $I = 365$ for ordinary years or $I = 366$ for leap years). This fundamental description enables best the understanding of the next chapters where primarily the difference between deterministic and stochastic components (e.g. deterministic vesrus stochastic trends as in Fatichi et al., 2009) has to be made. However, first, the types of analyzed data and the characteristics derived from them must be mentioned.

Furthermore, the term *stationarity* is frequently used throughout the text of this thesis and, therefore, it should be properly defined right at the beginning. According to Machiwal and Jha (2012), "stationarity implies that the statistical parameters of the series computed from different samples do not change except due to sampling variations". There are two types of stationarity usually distinguished by scientists. The first type is called *strict stationarity* that means the statistical properties of a series do not vary with changes of time origin. However, such conditions are not met in practice, so we will assume here what is called *weak stationarity*, which often suffices for practical purposes. It means that the first- and second-order moments are dependent only on time differences (e.g. Chen and Rao, 2002).

Nevertheless, there is also another term that should not be confused with stationarity and that is called *homogeneity* of a time series. It "implies that the data in the series belong to one population, and therefore have a time invariant mean" (Machiwal and Jha, 2012).

2. Data description and their preprocessing

2.1 Hydrometeorological data sources

Hydrology and climatology are typical of employing many people collecting the data and the analyst has no chance to control the entire process (including the so-called secondary processing). Moreover, the measurements last usually longer than a human life. Therefore, it is necessary to get the data from different sources. In ideal case, there are national services aimed at hydrology and climatology. In some countries, these services are split (i.e. one for hydrology and the other for climatology), but, fortunately, this is not the case in Czechia where the services are under the umbrella of one institute – **the Czech Hydrometeolorological Institute (CHMI)**.

Because the work performed during the author's Ph.D. study aimed primarily at the territory of Czechia, exactly the CHMI was the main source of data. However, the river basins investigated in some papers cross the national borders and thus mainly the climatological data had to be gathered from surrounding services. While in papers A7–A9 the additional source of precipitation totals in daily time step was the Polish service known as **the Institute of Meteorology and Water Management - National Research Institute (IMWM-NRI)**, for the purposes of papers A1 and A2 it was necessary to collect German data on precipitation, snow cover and air temperature from two bodies – national service called **Deutscher Wetterdienst (DWD)**, which since July 2014 has offered its data as downloadable text files from its website (http://www.dwd.de/), and **the Technical University in Dresden (TU Dresden)**, which, at the time of our request, maintained the database called Sächsische Klimadatenbank.

2.2 Types of data and the regions they represent

2.2.1 Hydrological data

Among the hydrological data that were analyzed in the papers (A1–A4) belong **discharge** series. They merely represented the Czech rivers observed at water-gauging stations of the CHMI. Although this variable integrates the processes occurring in river basins, it is in fact a derived variable that comes into being by relating the continually measured water level using the so-called rating curves, a graphical representation where water level is plotted against the *ad hoc* measured discharge and the estimation of fitting curve is made (see WMO, 2008). The influence of rating curves may play a crucial role in the detection of CC (Šercl, 2016, personal communication).

The discharge series were analyzed for the purposes of nation-wide comparison (144 water-gauges in A3 and A4) and in a more detailed way for the catchments of the Rolava River, located in the western Ore Mountains, NWW Bohemia, of the Vydra, Ostružná and Blanice rivers, located in the Bohemian Forest or its foothills, SW Bohemia, and of the Opava and Opavice rivers, located in the NE Czechia where they drain the Jeseníky Mountains (A1 and A2).

As regards the time span, the data in papers A3 and A4 represented the common hydrological period 1961–2005 (i.e. the series started 1 November 1960 and ended 31 October 2005). This dataset composed of daily data was prepared earlier by Fiala et al. (2010) and did not contain any missing values. The beginning of hydrological series in papers according to the tables 1 in papers A1 and A2 slightly ranged from 1931 (Modrava station on the Vydra River) to 1968 (Stará Role station on the Rolava River).

In paper A5, **spring-yield** series measured by the CHMI using predominantly calibrated buckets at least once per week were tested for the presence of regional trends. In particular, there were 157 well-springs from the entire territory of Czechia subjected to this analysis. The data were allowed to contain missing values since the method applied did not require the data to be complete. However, only some proportion of missing values was allowed to ensure not biasing the results too much (see Table 4 in paper A5). The common period which was represented by the data were the calendar years 1971–2007. The monthly spring yields (expressed as medians) were analyzed.

2.2.2 Climatological data

Here, the two basic groups of data should be distinguished

1. raw series accessed from the climatological services databases;
2. so-called technical reference series in daily time step based on (1), but filled and homogenized for the period at the Meteorology and Climatology Division of the CHMI and starting by the calendar year 1961.

Regarding the type-1 data, three variables measured by the climatologists of the CHMI were analyzed in papers A1 and A2 – **precipitation, snow cover depth and air temperature**, while in papers A7–A9 only precipitation was considered. The data from several precipitation stations (where snow cover is observed besides precipitation) and climatological stations were acquired. Their number and the area covered by them depended on the purpose of the study. Where spatial interpolation was needed, also the data from outside the river basins in the Ore Mountains, the Bohemian Forest and the Jeseníky Mountains were used. The original time step was daily but, finally, monthly time step sufficed for the majority of analyses performed in papers A1 and A2. Also the time coverage differed in this case but the hydrological period 1962–2008 was preferred due to the comparisons planned. Mainly the precipitation data from the Ore Mountains were somewhat gappy and had to be filled (details can also be studied in Kliment and Matoušková, 2009; Královec, 2009; Ledvinka, 2008).

In studies A7–A9, daily precipitation totals coming from the Polish and Czech rain-gauges situated within the upper part of the Lusatian Neisse River basin were assessed. The period of interest were the calendar years 1961–2010. Maximally 25% of missing values were allowed for each station that were then estimated due to the requirements.

Te type-2 data have been subjected to the analyses only in paper A6 so far. In particular, changes in precipitation totals at 268 stations covering the whole of Czechia were studied there. The calendar period 1961–2012 (but also others in a sequential manner) was investigated.

How exactly climatological data were measured in Czechia can be found in Tolasz (2007). Note, however, that in surrounding countries the measuring approaches may slightly differ. For instance, while in Czechia the mean daily air temperature $\overline{TMP}$ is computed according to

$$\overline{TMP} = \frac{TMP_{(7)} + TMP_{(14)} + 2\,TMP_{(21)}}{4} \tag{2.1}$$

where $TMP_{(\cdot)}$ represents air temperature observed at a certain hour (Central European Time, CET), in Germany, the climatologists changed this procedure in April 2001 to (e.g. Frick et al., 2014)

$$\overline{TMP} = \frac{TMP_{(0)} + TMP_{(1)} + TMP_{(2)} + \ldots + TMP_{(23)}}{24} \tag{2.2}$$

or

$$\overline{TMP} = \frac{TMP_{(0)} + TMP_{(6)} + TMP_{(12)} + TMP_{(18)}}{4} \tag{2.3}$$

when Universal Time Coordinated (UTC) is considered instead.

2.3 Derivation of other indicators

As already mentioned several times, not only the basic data but also the derived characteristics were analyzed. Sometimes, simple aggregation was sufficient. However, mostly new indices related to either hydrological drought or precipitation/rainfall extremes had to be computed. Moreover, in paper A6, changes in seasonality indices were evaluated.

2.3.1 Data aggregation

Perhaps the easiest preprocessing activity was the data aggregation. Even though in the lines above it is mentioned that the daily time step was the basic one, often their monthly counterparts were taken into account (see papers A1, A2, A5 and A6). In the case of discharge, air temperature and somewhat atypically also for snow cover depth, the monthly values represented the averages of daily ones. The same applied to the quarter-year (seasonal), semiannual and annual values. The precipitation series were aggregated using corresponding sums in place of means. Slightly different situation regarded spring yield were daily or weekly values were aggregated using medians to get monthly values first.

Particularly, in papers A1 and A2, monthly and annual series were subjected to the analyses. Note that a year was defined as the period between the month of November of the preceding calendar year and the month of October of the actual calendar year. This corresponds to a hydrological (water) year defined in Czechia. Snow cover depth variable deserves special attention which, of course, has nonzero values only for the winter periods under the conditions in Czechia. Therefore, the year for this variable was represented by the months between November of the preceding calendar year and May of the actual calendar year.

In papers A5 and A6, months and calendar years (not the hydrological ones) were investigated. Moreover, in paper A5, seasonal (spring from March to May, summer from June to August, autumn from September to November, and winter from December to February) and semiannual (warm half-year from May to October, cold half-year from November to April) data were derived for the next analyses.

2.3.2 Orographic spatial interpolation of the CHMI

In hydrology, often it is necessary to get the information about precipitation over the whole river basin and not only for the stations where the measurements are carried out for economic reasons. It is apparent notably when the investigator wants to compare the amount of precipitation with the flow rate at some closing profile of the basin. Also, this would be valuable when setting a stochastic model incorporating external variables such as precipitation and other elements relating to climate.

In papers A1 and A2, the so-called *orographic interpolation* technique developed at the CHMI was employed when the series of areal precipitation (sometimes called rainfall depth) for each month were necessary. In fact, the second generation of this prepared tool was used that takes into account the relationships between precipitation and altitude represented by a digital elevation model (amounts typically rise with altitude; Šercl, 2016, personal communication). Because the tool is not the product of the author and the GIS applications are beyond the scope of the thesis, the interested readers are kindly referred to the paper by Šercl (2008).

2.3.3 Hydrological drought indices

Papers A3 and A4 are exclusively devoted to the investigation of hydrological drought over the territory of Czechia. For this purpose, several indices were derived. Using daily discharge data, as in the literature traditionally, two types of indices were assessed

1. Characteristics derived with the aid of moving averages. For example, the 7-, 15- and 30-day moving averages are first computed and then the minima corresponding to some season (e.g. entire hydrological year or summer and winter periods) are selected. The timing of these minima is also often studied (see e.g. Ehsanzadeh and Adamowski, 2010; Khaliq et al., 2008). In papers A3 and A4, only the 7-day low flows and their timing were analyzed this way, although 15- and 30-day counterparts were prepared for future works.

2. Characteristics produced by the threshold level method (TLM), where a threshold corresponding to a sufficiently small quantile of the series is first chosen. Although in the world literature, quantiles are calculated for the probability of exceedance given in percents (say 90%), in Czechia hydrologists prefer to use the so-called M-day discharges corresponding to the prob-

ability that the discharge is, on average, exceeded during M days throughout the year (i.e. $M = 330$ or $M = 355$ as in paper A4; COSMT, 2014). Integrating the values of discharge lacking to satisfy the condition of being at least equal to the value of the threshold gives the so-called deficit volume. The number of days with the discharge below the threshold is often called the drought duration. To form the series of these characteristic one may again focus on different time periods such as summer, winter or the whole year, as done in papers A3 and A4.

Note that methods dealing with the short excesses over the threshold do exist that may slightly change the resulting series of deficit volume and drought duration (see e.g. Tallaksen and van Lanen, 2004) but they were not applied because the main goal of papers A3 and A4 was the consistency with the outcomes of Fiala et al. (2010). Note again that solely 7-day low flows were studied in the papers and the incorporation of the latter two moving averages is foreseen. Besides these characteristics, a lot of other (some of them currently being new ones) based on monthly time series can be found in the literature. Their development is in line with the attempts to find a good signal of large-scale climate patterns in water resources (see van Huijgevoort et al., 2012). This, however, was not the objective of the thesis.

Furthermore, the studies A3 and A4 differ from the rest in specifying the hydrological years. Namely, 1 April was set as the beginning of such a year because of the hydrological droughts timing in Czechia. It may pose some issues when working with Julian days closely associated with the timing in trend analysis. Thus, hydrologists (e.g. Fiala et al., 2010) sometimes transform the Julian days JD_i to angular units Ω_i ($0 \leq \Omega \leq 2\pi$) according to (I being the number of days in a year)

$$\Omega_i = JD_i \frac{2\pi}{I} \tag{2.4}$$

which then can be shifted by the desired three months by subtracting $\pi/2$ from the right-hand side of the Eq. 2.4. After that, a trend analysis may be applied properly to the series of Julian days. However, it was not the case of papers A3 and A4 where special functions of the `lubridate` R package (Grolemund and Wickham, 2011) were used instead, which yielded satisfactory approximation.

In papers A3 and A4, also the seasons investigated deserve their attention. The summer season was defined by the months from April to November, while the winter season by the months from December to March.

2.3.4 Precipitation characteristics and the number of days with snow cover

Paper A6 deals with the precipitation patterns observed at 268 stations over the territory of Czechia. Apart from the precipitation totals as such (and aggregated as well), there were two indices constructed according to Brunetti et al. (2001). These were the annual series of numbers of days with nonzero precipitation (so-called wet days, WD). Given the annual precipitation totals R, also the annual precipitation intensity IN could be computed for each k-th year as follows

$$IN_k = \frac{R_k}{WD_k} \tag{2.5}$$

In papers A7–A9, two types of series corresponding to precipitation/rainfall maxima had to be derived from the daily data prior to the analyses

1. Data acquired by the peaks-over-threshold (POT) method which is similar to the TLM mentioned above. The difference is that maxima (peaks) are selected instead of minima and we are interested only in independent occurrences so we could study different rainfall storms. The term *rainfall* is intentionally used here, because the goal was to separate the warm season from the cold season of the year in which different climate drivers usually stand behind precipitation maxima. According to the experience of Polish colleagues, the criterion of one day was chosen that should have ensured the independence among data. Note that a fixed threshold of 95% (from annual nonzero precipitation) was selected in papers A7 and A9, while in paper A8 the thresholds varied according to the criteria described in Gilleland and Katz (2014). Also, there was another difference between papers A7 and A9 on one hand and paper A8 on the other. Namely only 50 largest POT values were used in papers A7 and A9, while all the POT values were involved in the analysis in paper A8

2. Annual maximum series composed simply of the largest values observed during each year. This approach can also be formalized. Having K years and $i = 1, 2, \ldots, I$ days within each year, the selection of maxima x_k may be written as
$$x_k = \max_i \{x_{i,k}\} \tag{2.6}$$

In fact, POT series *per se* do not allow a pertinent trend analysis. Therefore, a test was applied to the values corresponding to the quantiles computed for each year using the following plotting position

$$p(o, I) = \frac{o}{I+1} \tag{2.7}$$

where o denotes the element of the variational series of I nonzero daily precipitation totals pertaining to an investigated season. Although this plotting position is considered somewhat inappropriate (Hyndman and Fan, 1996), the reason for using it was the consistency with Polish colleagues who employ it quite a lot (see e.g. Kaźmierczak and Kotowski, 2015). Similarly, trends in numbers of POT values in each year were identified.

Furthermore, papers A1 and A2 contain sections where the series composed of the numbers of days with snow cover were analyzed. These series were formed in a similar manner to the series of days with nonzero precipitation in paper A6.

2.3.5 Seasonality indicators and deseasonalization

In paper A6, some interesting aspects regarding seasonality are studied. Motivated by the paper of Feng et al. (2013) and using the monthly precipitation data $r_{k,m}$ with monthly index $m = 1, 2, \ldots, 12$, the author derived three indicators whose series were examined for trends

- seasonality centroid (also referred to as seasonality timing) C_k

$$C_k = \frac{1}{R_k}\sum_{m=1}^{12} m r_{k,m} \tag{2.8}$$

with R_k defined above as an annual total.

- seasonality spread (also referred to as seasonality duration) Z_k

$$Z_k = \sqrt{\frac{1}{R_k}\sum_{m=1}^{12}(m - C_k)^2 r_{k,m}} \tag{2.9}$$

- seasonality index SI_k

$$SI_k = R_k \cdot \sum_{m=1}^{12} p_{k,m}\log_2(12p_{k,m}) \tag{2.10}$$

where

$$p_{k,m} = \frac{r_{k,m}}{R_k} \tag{2.11}$$

The sum in the seasonality index is called entropy. Sometimes, it may happen that there is no precipitation even during the whole month. In this case the following limit was used and introduced into Eq. 2.10 similarly as in Sohoulande Djebou et al. (2014)

$$\lim_{p_{k,m}\to 0+} p_{k,m}\log_2(12p_{k,m}) = 0 \tag{2.12}$$

Seasonality present in a time series imply that the independence among data is violated. Therefore such a series must be deseasonalized before the application of some tests. In papers A1 and A2, this was not necessary because the authors worked with the monthly time series separately. However, in papers A5 and A6, different deseasonalization procedures had to be conducted because in paper A6 the deseasonalization process was applied to daily data, whereas in paper A5 to monthly data.

Hydrologists prefer a simple deseasonalization technique sometimes called seasonal standardization dealing with seasonality in the mean and variance only. Thus, higher-order moments as well as autocorrelation are not addressed. In paper A5, it was also the case where monthly spring-yield series were deseasonalized similarly as in Fatichi et al. (2009); Grimaldi (2004); Kantelhardt et al. (2006) or Salas (1993). This can be formalized in matrix notation.

Consider, for convenience, that a monthly time series x_t is organized in a matrix $\mathbf{X}$ as follows

$$\mathbf{X} = [x_{k,m}] = \begin{bmatrix} x_{1,1} & x_{1,2} & \dots & x_{1,12} \\ x_{2,1} & x_{2,2} & \dots & x_{2,12} \\ \vdots & \vdots & \ddots & \vdots \\ x_{K,1} & x_{K,2} & \dots & x_{K,12} \end{bmatrix}$$

Now, defining the K by K matrix $\mathbf{1}$ whose elements are all equal to 1, the matrix $\mathbf{X}'$ with seasonally centred data can be obtained by

$$\mathbf{X}' = \mathbf{X} - \frac{1}{K}\mathbf{1X} \tag{2.13}$$

Then, having the 12 by 12 matrix $\mathbf{D}$ whose main diagonal is composed of standard deviations corresponding to respective months (and other elements are equal to zero), the matrix $\mathbf{X}^{DS}$ with deseasonalized data can be computed as

$$\mathbf{X}^{DS} = \mathbf{X}'\mathbf{D}^{-1} \tag{2.14}$$

Another deseasonalization was applied to daily precipitatin series in paper A6. Since this approach was based on wavelet analysis and it is not the main subject here, the readers are referred to its authors Szolgayová et al. (2014). More theoretical background on wavelets can be found in Daubechies (1992). Theoretical details of the wavelet methods suitable for time series analysts are described in Percival and Walden (2000).

3. Statistical methods applied in the attached papers and more

Here, a description of the majority of the methods applied by the author in papers A1 to A9 is summarized regardless whether the techniques were already described in detail or not there. The meaning of this is to transfer the whole key methodology to one place and thus to allow for a better orientation. Also, unlike in the papers, a unified notation is used. When appropriate, also some additional methods are introduced due to the illustration or comparison purposes or owing to the fact that they were applied by the author in other published material that is not included here. On the other hand, methods whose detailed description was beyond the scope of the thesis are omitted. At the same time, the author admits that some methods would definitely require more detailed discussion. However, rather, it was postponed to the general discussion in Chapter 5.1.

3.1 Data completion techniques

Although data infilling could have been addressed in the chapters devoted to data preprocessing, this topic was rather placed here since it is quite interesting from a statistical point of view. Here exclusively, the precipitation data are addresses because no other data had to be filled in throughout the papers. Regarding precipitation, during the past, it happened that the measurement at a station began later or finished earlier than it would be desirable for hydrologists dealing with CC and calling for longer series. Also, unwanted moving of measuring sites occurs. As a result of this, the precipitation series suffer from incompleteness for a given place that poses several issues to statisticians whose methods are predominantly designed for uninterrupted series. Therefore, the missing values are rather estimated by various techniques prior to the main analyses.

3.1.1 Method of quotients

Working predominantly on the Rolava River basin in papers A1 and A2, before spatial interpolation and the preceding imputation of missing precipitation values using a more rigorous method, the author initialized the estimation via the method of quotients outlined, for example, in Conrad (1946). Having several monthly precipitation series r_t each split according to the 12 months in 12 series r_k, the station A with a longer observed series of K_1 elements and a good proximity to the station for which the process of imputation has to be made (e.g. a, with length of observations K_2) may serve as a good donor of missing values estimates if there is a considerable overlap of observations. From the overlapping period, the so-called quotient q_0 is computed first according to

$$\frac{{}_a\overline{r}_{K_2}}{{}_A\overline{r}_{K_2}} = q_0 = \frac{{}_a\overline{r}_{K_1}}{{}_A\overline{r}_{K_1}} \tag{3.1}$$

where the long-term total ${}_a\overline{r}_{K_1}$ is unknown. However, it can be calculated as

$$ {}_a\overline{r}_{K_1} = q_0 \cdot {}_A\overline{r}_{K_1} \tag{3.2} $$

Obviously, not only the long-term totals for the particular months can be computed this way. The method can be extended for any element $\hat{r}_k$ pertaining to a particular year if the quotient and longer series, which fulfills the condition of overlap, are available. On the other hand, this method cannot be used for exaggerated extrapolations if CC is taking the effect because in Eq. 3.1 it is assumed that the ratios are quasi-constant. Therefore, only short sequences were filled in where possible before the main analyses in papers A1 and A2.

3.1.2 A technique based on multiple linear regression

Multiple linear regression (MLR) is a widely used tool and generally known from fundamental statistical courses (e.g. Kottegoda and Rosso, 2008). The whole infilling process for the Rolava River basin in papers A1 and A2 comprised nine steps (e.g. series merging, excluding erroneously recorded data; Ledvinka, 2008) but its final steps relied just on MLR which is capable of estimating theoretical values $\hat{r}_k$ based on the relationships among series.

After dividing monthly time series with regard to the 12 months (e.g. Januaries, Februaries, etc.) and S stations, the full data (i.e. complete time series pertaining to one month) can form a matrix with regressors $\mathbf{X}$

$$ \mathbf{X} = [x_{k,s}] = \begin{bmatrix} x_{1,1} & x_{1,2} & \dots & x_{1,S} \\ x_{2,1} & x_{2,2} & \dots & x_{2,S} \\ \vdots & \vdots & \ddots & \vdots \\ x_{K,1} & x_{K,2} & \dots & x_{K,S} \end{bmatrix} $$

At the same time, there is a series that contains missing values which need to be estimated. This time series can be represented by a vector $\mathbf{y}$

$$ \mathbf{y} = [y_k] = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_K \end{bmatrix} $$

whose elements can be missing at some places. Then, the MLR model may be written as

$$ \mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\eta} \tag{3.3} $$

where $\boldsymbol{\eta}$ is a column vector of K elements corresponding to random errors, each of which is assumed to be independent and identically (i.e. zero-mean Gaussian) distributed ($\eta_k \sim iid \quad \mathcal{N}(0, \sigma_\eta^2)$), and $\boldsymbol{\beta}$ is an unknown column vector of regression coefficients of order S equal to the number of stations whose series should serve as explanatory variables.

Often, the vector $\boldsymbol{\beta}$ is estimated using the ordinary least squares (OLS) method where one attempts to minimize the following expression

$$ (\mathbf{y} - \boldsymbol{\beta}\mathbf{X})^\top (\mathbf{y} - \boldsymbol{\beta}\mathbf{X}) $$

where the symbol $\top$ denotes the transpose of a matrix. Taking the partial derivatives with respect to each element of the vector $\boldsymbol{\beta}$ and putting them equal to zero yields the so-called normal equations that can be used for obtaining the estimate of $\boldsymbol{\beta}$. This process may be formalized in matrix notation (e.g. Haan, 2002; Kottegoda and Rosso, 2008; Shahin et al., 1993; Wilks, 2011)

$$\hat{\boldsymbol{\beta}} = (\mathbf{X}^\top\mathbf{X})^{-1}\mathbf{X}^\top\mathbf{y} \tag{3.4}$$

Using $\hat{\boldsymbol{\beta}}$, the modelled values $\hat{\mathbf{y}}$ may be computed as

$$\hat{\mathbf{y}} = \mathbf{X}\hat{\boldsymbol{\beta}} \tag{3.5}$$

by which the gaps can be filled in $\mathbf{y}$.

Of course, Eq. 3.4 must be updated according to the places of missing values and overlaps of existing values. Exactly this had to be done in the thesis of Ledvinka (2008) which preceded papers A1 and A2. Note also that Ledvinka (2008) applied a criterion where at least 15 years of explanatory variables had to be measured simultaneously with the series for which the imputation process was needed.

3.1.3 Data infilling implemented in the `hyfo` `R` package

In papers A8 and A9, where the Czech data were added to the Polish data, a quick approach to fill in the missing values in daily precipitation series was necessary before the analyses themselves. The function `fillGap()` that can be found within the `hyfo` `R` package (Xu, 2016) was a good candidate at the time of papers preparation.

The procedure starts with the computation of a matrix $\mathbf{C}$ containing the estimates of Pearson moment correlation coefficients (PMCCs) c between each pair of precipitation stations. That is

$$\mathbf{C} = \operatorname{corr}(r_s^{(1)}, r_s^{(2)}) = [\hat{c}_{s,s}] = \begin{bmatrix} \hat{c}_{1,1} = 1 & \hat{c}_{1,2} & \dots & \hat{c}_{1,S} \\ \hat{c}_{2,1} & \hat{c}_{2,2} = 1 & \dots & \hat{c}_{2,S} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{c}_{S,1} & \hat{c}_{S,2} & \dots & \hat{c}_{S,S} = 1 \end{bmatrix}$$

with (1) and (2) indicating rows and columns, respectively, and $\hat{c}_{s,s}$ calculated as

$$\hat{c}_{s,s} = \frac{\operatorname{cov}(r_s^{(1)}, r_s^{(2)})}{\sqrt{\operatorname{var}(r_s^{(1)})\operatorname{var}(r_s^{(2)})}} \tag{3.6}$$

where

$$\operatorname{cov}(r_s^{(1)}, r_s^{(2)}) = \frac{1}{T-1}\sum_{t=1}^{T}(r_{s,t}^{(1)} - \mu_{r_s^{(1)}})(r_{s,t}^{(2)} - \mu_{r_s^{(2)}}) \tag{3.7}$$

is a covariance between a pair of precipitation series (of T observations in general) with $\mu_{r_s^{(\cdot)}}$ being the respective means of the series, and

$$\operatorname{var}(r_s^{(\cdot)}) = \frac{1}{T-1}\sum_{t=1}^{T}(r_{s,t}^{(\cdot)} - \mu_{r_s^{(\cdot)}})^2 \tag{3.8}$$

is a variance defined generally either for a row or a column variable $r_s^{(\cdot)}$.

Then, another matrix $\mathbf{B}$ composed of the estimates of linear regression coefficients β that correspond to the above correlation coefficients and satisfy the condition

$$r_s^{(1)} = \beta r_s^{(2)} + \iota, \quad \iota = 0, \quad s = 1, 2, \ldots, S \tag{3.9}$$

is created, so

$$\mathbf{B} = [\hat{\beta}_{s,s}] = \begin{bmatrix} \hat{\beta}_{1,1} = 1 & \hat{\beta}_{1,2} & \ldots & \hat{\beta}_{1,S} \\ \hat{\beta}_{2,1} & \hat{\beta}_{2,2} = 1 & \ldots & \hat{\beta}_{2,S} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\beta}_{S,1} & \hat{\beta}_{S,2} & \ldots & \hat{\beta}_{S,S} = 1 \end{bmatrix}$$

where $\hat{\beta}_{s,s}$ are computed as follows

$$\hat{\beta}_{s,s} = \frac{\overline{r_s^{(1)} r_s^{(2)}}}{\overline{r_s^{(2)2}}} \tag{3.10}$$

where horizontal bars mean averages of the product in the nominator and the averages of squares of the precipitation series in the denominator, at the same time representing columns of matrix $\mathbf{B}$, respectively.

According to the values of correlation coefficients (the higher, the better; excluding unities), the best relationship is selected for gapfilling. When the highest correlation coefficient for a given precipitation series $r_s^{(1)}$ is found in the matrix $\mathbf{C}$, then the corresponding regression coefficient is used together with the associated series $r_s^{(1)}$ in Eq. 3.9. While the series $r_s^{(1)}$ may contain missing values, the series $r_s^{(2)}$ may not. If the series $r_s^{(2)}$ contains the missing value for the same time index, the second highest correlation coefficient from the matrix $\mathbf{C}$ is searched. The procedure is repeated until the whole data set is complete, if possible (for more details see Xu, 2016).

3.1.4 Data infilling based on the triangular irregular network

The above two methods do not take into account the geographical distance and are merely based on the statistical relationships. In paper A7, a method that makes use of the geographical distances was applied when estimating missing values in daily precipitation series for Polish stations. According to the findings of Polish colleagues (Wdowikowski, 2016, personal communication), the technique works quite well in Poland, which was confirmed by the so-called precipitation double aggregation curves (PDAC) in paper A7. However, to the author's knowledge, it has not been applied in Czechia.

This method is similar to the inverse distance weighting technique frequently used for spatial interpolation in geography and elsewhere. The weights, however, are given differently here. First, it is necessary to create the triangles whose vertices are represented by the locations of stations. For illustration, the situation is depicted in Fig. 3.1 using four stations. In the middle, there is a station P_x for which the missing value have to be estimated. All the neighbouring stations have their own observed data at this time. As can be seen from Fig. 3.1, the weights

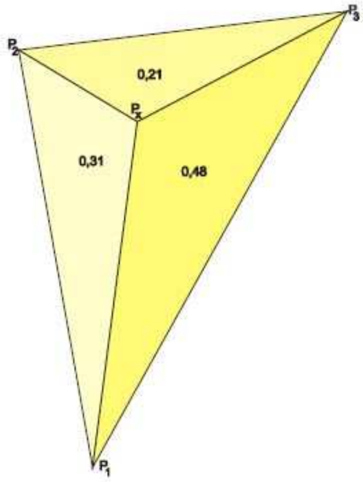

Figure 3.1: An example of constructed triangular irregular network before estimation of a missing value for the station in the middle using information from neighbouring stations (adapted from Sczepanek, 2003)

are given by the areas on the opposite sides of the vertices (in fact precipitation totals at some time) $P_1, \ldots, P_3$. Note that the sum of the weights is equal to 1. The missing value can then be estimated as

$$P_x = P_1 w_1 + P_2 w_2 + P_3 w_3 \tag{3.11}$$

where $w_1, \ldots, w_3$ are the weights corresponding to Ps with the same subscripts. Hence, for the example depicted in Fig. 3.1, the result is (Sczepanek, 2003)

$$P_x = P_1 0.21 + P_2 0.48 + P_3 0.31$$

The trianguar model must be updated if the network of stations changes due to stations shifts or instrumental malfunctions, which can make some difficulties when incorporating this method in software.

3.1.5 A note on the need for data infilling

A question remains if the infilling process is necessary at all. Definitely, it is if the main objective of a study is to investigate the properties of the time series measured at stations. For example, the works such as Teegavarapu (2014a,b) and many other papers of the same author illustrate how important this issue is and to what extent the techniques devoted to it evolved.

However, in hydrology, often the data related to climate are spatially interpolated (as was done in papers A1 and A2) prior to their comparisons with hydrological data representing areas such as river basins. Of course, different sources of uncertainty have to be taken into consideration, some of them being closely connected with interpolation techniques themselves. Also the time step of time series under consideration plays a crucial role (see e.g. Ly et al., 2013).

Therefore, sometimes it would be better to interpolate a variable and sometimes not and rather prefer to study a time series coming from a near meteorological station. Since there is no universal rule, it should certainly be studied more carefully before the initiation of TSA.

Another issue is that none of the above methods accounts for the autocorrelation structure or lagged cross-correlation structure of time series. Such models have a great potential in data imputation process. And they certainly exist but the frequency domain is somewhat preferred in geophysics (Kondrashov and Ghil, 2006; v. Buttlar et al., 2014). However, their employment must have been postponed due to the lack of knowledge of Czech hydrologists in general.

3.2 Abrupt changes detection in general

Despite gradual changes are usually searched in hydrometeorological series regarding CC, also abrupt (step) changes are important. These changes are caused rather by a phenomenon that occurs suddenly which may be of anthropogenic or natural origins (e.g. after disturbances caused by floods). Anthropogenic impacts detected through time series can be of two types

- direct impacts in the environment;
- erroneous measurement or subsequent processing of data (e.g. rating curves through which discharges are derived from measured water levels; Šercl, 2016, personal communication).

For the second type of anthropogenic changes, especially in climatology, the homogeneity tests were developed (see e.g. Conrad, 1946). However, these tests may be used also for the detection of the first-type anthropogenic impacts and the changes caused by nature alone (Kundzewicz and Robson, 2000, 2004; Machiwal and Jha, 2012). These changes are usually detected in a univariate fashion but in recent years also tests making use of more than one series have been developed, specifically those that need the metadata confirming the time of change.

Because in papers A1 and A2 some of these tests were applied and they were not thoroughly described there, a need for their detailed description arose at this place. Here, a general annual time series x_k will be considered since this topic includes not only precipitation but also other variables.

3.2.1 Wilcoxon rank sum test

This test is designed for situations when the analyst has a time series x_k that is suspected of being composed of two parts apparently breaking at some time that is considered known. The parts have different lengths, that is, the longer one $x_k^{K_1}$ has the length K_1 and the shorter one $x_k^{K_2}$ has the length K_2 (thus $K_1 + K_2 = K$), so it is not possible to pair the data. The analyst wants to confirm the time of change between these two parts. Setting the null hypothesis

$H_0 : \Pr(x_k^{K_1} > x_k^{K_2}) = 0.5$ (Helsel and Hirsch, 2002), the five steps are performed (Hirsch et al., 1993)

1. Assign ranks to the elements of the series from 1 (smallest) to K (largest). When there are some ties present in the data, the averages are used for the ranks RNK for the tied values.

2. Compute the test statistic W as follows

$$W = \sum_{k=1}^{K_2} RNK_k \tag{3.12}$$

3. Because the approximation of the distribution of the statistic W is conducted using the standard Gaussian distribution in most of the cases, we need its theoretical mean

$$\mu_W = \frac{K_2(K+1)}{2} \tag{3.13}$$

and variance

$$\sigma_W^2 = \frac{K_2 K_1}{K(K-1)} \sum_{k=1}^{K} RNK_k^2 - \frac{K_2 K_1 (K+1)^2}{4(K-1)} \tag{3.14}$$

4. The standardized form of the test statistic Z_W is calculated as

$$Z_W = \begin{cases} \dfrac{W - 0.5 - \mu_W}{\sqrt{\sigma_W^2}} & \text{if } W > \mu_W \\ 0 & \text{if } W = \mu_W \\ \dfrac{W + 0.5 - \mu_W}{\sqrt{\sigma_W^2}} & \text{if } W < \mu_W \end{cases} \tag{3.15}$$

5. The statistic Z_W is then compared with the quantiles $u_{1-\alpha/2}$ of the standard Gaussian distribution where α is the prescribed level of significance. Considering the two-sided version of the test, if $|Z_W| > u_{1-\alpha/2}$, the test rejects H_0 which means that the change point is found.

According to Hirsch et al. (1993), the mentioned approximation can be used if $\min\{K_1, K_2\} \geq 10$. Otherwise, the exact test, where the distribution of the statistic W is accounted for, must be applied.

3.2.2 Kruskal–Wallis test

When the analyst suspects that there are more than one change points, the Kruskal–Wallis test may be used for finding if at least one section of a time series differ in distribution from the rest. The following four steps are performed regarding this test (Hirsch et al., 1993)

1. As was the case for the preceding test, assign ranks to data.

2. For each of the g sections, calculate the average rank $\overline{RNK}_j$ for each of the k_j data values pertaining to that section. So

$$\overline{RNK}_j = \frac{1}{k_j} \sum_{k=1}^{k_j} RNK_k \tag{3.16}$$

3. The test statistic KW is as follows

$$KW = \frac{12}{K(K+1)} \sum_{j=1}^{g} k_j \left(\overline{RNK}_j - \frac{K+1}{2} \right)^2 \tag{3.17}$$

4. The null hypothesis H_0 is rejected if $KW \geq \chi^2_{1-\alpha,\nu}$ (the quantile of the χ^2 distribution), where $\nu = g - 1$ are the degrees of freedom which the shape of the χ^2 distribution is dependent on.

Again, there is a need to use exact test when the sample size is realy small (say ≤ 5 observations in each of 3 groups, or ≤ 4 observations in each of ≥ 4 groups).

3.2.3 Pettitt test

The above two tests rely on the assumption that we, more or less, know the location of the change point(s). However, in hydrology there are situations when the analyst was not given such information in advance (e.g. metadata are missing). In that case Kundzewicz and Robson (2000, 2004); Kundzewicz and Radziejewski (2006) recommend the Pettitt test (also known as the Pettitt–Mann–Whitney (PMW) test; Pettitt, 1979) that was also applied in papers A1 and A2 to the precipitation and discharge series.

The null hypothesis says that the medians of different part of a series are the same. The test statistic PT is defined for each time and is computed as

$$PT = \max_{1 \leq k \leq K} \left| \sum_{\ell=1}^{k} \sum_{j=k+1}^{K} \operatorname{sgn}(x_\ell - x_j) \right| \tag{3.18}$$

Whether the H_0 holds true or not can be evaluated by the p-value that is (for $p \leq 0.5$; see Pettitt, 1979) frequently approximated by

$$p \simeq 2 \exp \left(\frac{-6PT^2}{K^3 + K^2} \right) \tag{3.19}$$

or by comparing PT with a critical value dependent on the sample size K and the prescribed level of significance α, that is a threshold value $PT_{\text{th},K}$

$$PT_{\text{th},K} = \sqrt{\frac{-\ln \alpha \, (K^3 + K^2)}{6}} \tag{3.20}$$

The test evidently tends to indicate change points at the beginning and the end of a series too often and thus sometimes the search is restricted to a middle part (say the section in between 10 values after the start and 10 values before the end) of the series only (see e.g. Kyselý and Domonkos, 2006). However, in papers A1 and A2 the whole time series were examined.

3.2.4 Standard normal homogeneity test (SNHT)

The SNHT in its basic form (Alexandersson, 1986; Alexandersson and Moberg, 1997) is similar to the PMW test but the test statistic is defined differently. First, the standardized values z of the tested series x are computed. That is

$$z = \frac{x - \overline{x}}{s_x} \tag{3.21}$$

where, repeating rather once again, $x = \{x_1, x_2, \ldots, x_K\}$ is the original time series and $z = \{z_1, z_2, \ldots, z_K\}$ is its standardized counterpart; moreover, s_x is the standard deviation of the series x. Then, the null hypothesis H_0 is stated as follows (Alexandersson and Moberg, 1997)

$$H_0 : z_k \in \mathcal{N}(0, 1) \quad k \in \{1, 2, \ldots, K\}$$

while the alternative hypothesis H_1 as

$$H_1 : \begin{cases} x_k \in \mathcal{N}(\mu_1, 1) & k \in \{1, \ldots, \delta\} \\ x_k \in \mathcal{N}(\mu_2, 1) & k \in \{\delta + 1, \ldots, K\} \end{cases}$$

where δ is some unknown time when the change in mean occurred.

Alexandersson (1986) developed the following test statistic

$$T^S_{\max} = \max_{1 \le \delta \le K-1} T^S_\delta = \max_{1 \le \delta \le K-1} \{\delta \overline{z}_1^2 + (K - \delta)\overline{z}_2^2\} \tag{3.22}$$

with arithmetic averages of values before the change ($\overline{z}_1$) and after the change ($\overline{z}_2$). The statistic has its own distribution dependent on the sample sizes for which the critical values (for various α) are tabulated. For instance, Khaliq and Ouarda (2007) used Monte Carlo simulation for this purpose. Exactly this way the series of air temperature and snow cover depth were tested for the presence of change points in papers A1 and A2.

As with the PMW test, the SNHT suffer from the drawback that it indicates too often the change points at the beginning and at the end of the series. This, again, lead some authors to restrict the investigated section of the series to its middle part (see e.g. Kyselý and Domonkos, 2006; Štěpánek, 2004).

3.2.5 Von Neumann's ratio

Despite not being stated in the attached papers, the author of the thesis used also von Neumann's ratio for checking the homogeneity of the series that were in fact composed of two merged series coming from precipitation stations with very close proximity. This control (for the Nejdek precipiation station in particular) was perfomed in diploma thesis (Ledvinka, 2008) before the data imputation process started regarding the Rolava River basin. This test does not give the time of change, it only says whether a series is homogeneous or not.

The test statistic is defined as follows (Buishand, 1982)

$$VN = \frac{\sum_{k=1}^{K-1} (x_k - x_{k+1})^2}{\sum_{k=1}^{K} (x_k - \overline{x})^2} \tag{3.23}$$

with $\overline{x}$ again indicating the average of the series x_k.

For homogeneous samples $VN = 2$. For the series containing a break, this statistic tends to be lower. The critical values for various significance levels α and sample sizes are usually tabulated. For the most used $\alpha = 0.05$ or $\alpha = 0.01$, the critical values can be found in Wijngaard et al. (2003).

Note that there may be the cases with $VN > 2$. This situation is caused by rapid variations in the mean of the series (Wijngaard et al., 2003).

3.3 Relative abrupt changes detection

Sometimes it is necessary to find out if there are changing patterns in more than one series measured simultaneously. For example, one may test if the relationship between two time series (e.g. precipitation versus discharge) somewhat change (i.e. if after some time one series tends to have significantly greater values than the other). In that case, the paired versions of some popular tests such as the parametric t-test are frequently employed. Hydrologists and climatologists, however, prefer to use nonparametric counterparts. The first of the two described here will be the Wilcoxon signed rank test (applied in paper A9), while the second the Alexandersson test. Note that the author of the thesis respect the terminology used in Štěpánek (2004) in an attempt to distinguish the latter test from the SNHT. Otherwise the Alexandersson test is referred to as the SNHT as well in scientific literature.

3.3.1 Wilcoxon signed rank test

Assume we have two simultaneously measured series x_k and y_k. The null hypothesis of the Wilcoxon signed rank test H_0 states that both the series come from the same population, not necessarily Gaussian or symmetrically distributed. The alternative hypothesis H_1, on the contrary, states that the series differ in their level. When performing this tests, the steps are following (Hirsch et al., 1993)

1. Excluding the tied data (i.e. those where $x_k = y_k$), define n as the number of nonzero differences $D_k = x_k - y_k$.

2. Rank D_k according to their absolute values form the smallest $|D_k|$ with rank 1 to the largest $|D_k|$ with rank n. Again, the ranks will be assigned RNK_k.

3. Compute R^+ so that
$$R^+ = \sum_{D_k > 0} RNK_k \tag{3.24}$$

4. Compute R^- so that
$$R^- = \sum_{D_k < 0} RNK_k \tag{3.25}$$

5. Then, the test statistic W^+ is defined as the minimum of the two
$$W^+ = \min\{R^+, R^-\} \tag{3.26}$$

6. Under H_0, the mean of W^+ is calculated as

$$\mu_{W^+} = \frac{n(n+1)}{4} \tag{3.27}$$

while its variance as

$$\sigma^2_{W^+} = \frac{n(n+1)(2n+1)}{24} \tag{3.28}$$

7. If $n \geq 15$, the approximation based on the standard Gaussian distribution may be used. In that case, the statistic Z_{W^+} is computed according to

$$Z_{W^+} = \begin{cases} \dfrac{W^+ - 0.5 - \mu_{W^+}}{\sqrt{\sigma^2_{W^+}}} & \text{if } W^+ > \mu_{W^+} \\ 0 & \text{if } W^+ = \mu_{W^+} \\ \dfrac{W^+ + 0.5 - \mu_{W^+}}{\sqrt{\sigma^2_{W^+}}} & \text{if } W^+ < \mu_{W^+} \end{cases} \tag{3.29}$$

8. The hypothesis H_0 is rejected if $|Z_{W^+}| > u_{1-\alpha/2}$, where $u_{1-\alpha/2}$ is $1-\alpha/2$ quantile of the standard Gaussian distribution with α being the prescribed level of significance.

If $n < 15$, the exact test based on the real distribution of the statistic W^+ must be used.

It is worth noting that this test may also be utilized for detecting the abrupt/step changes in a single series when it can be split into two halves (one older and the second newer). The differences D_k then may be computed between these two halves as was done in paper A9 where the authors were inspired be the similar utilization of the paired t-test in Lettenmaier (1976).

3.3.2 Alexandersson test

In fact, the Alexandersson test is a modified version of the SNHT designed for two time series. Again, let x_k and y_k denote the two series compared. The procedure is the same as in the case of the SNHT described in Chapter 3.2.4. The exception is that prior to the application of the SNHT, the differences similar to those in that chapter (but this time not excluding $D_k = 0$) or quotients $q_k = x_k/y_k$ or $q_k = y_k/x_k$ are computed.

Note that, for instance, in the case of air temperature the differences are preferred, and in the case of precipitation, the quotients are often used. For discussion about the proper selection between differences and quotients, the readers are kindly referred to Conrad (1946).

3.4 A note on homogenization and RHtests

As was already said, the above tests are often used for checking the homogeneity. Most of them are designed for annual data (or annual data pertaining to one month such as January or February series). After finding inhomogeneity in a series, there are techniques based on which one can homogenize such a series.

The state-of-the-art homogenization of climatological time series in Czechia was described in detail by Štěpánek et al. (2012). This book also explains how the technical reference precipitation series analyzed in paper A6 came into being. To this publication, the papers of, more or less, the same authors written in English may be added as well (e.g. Štěpánek et al., 2011, 2013). However, this kind of homogenization was not carried out in any of the attached papers.

On the contrary, a methodology (RHtests) does exist that focuses exactly on daily (precipitation) data. It was utilized in papers A8 and A9 when we needed homogeneous rainfall series for our analyses of daily maxima. Since the methodology of homogenization was not the objective of this thesis and as such it is quite cumbersome, the reader is referred to Wang et al. (2010), who also created a script implementing it in `R`. Nevertheless, this homogenization was not performed in paper A7 because it was too late for it when the Polish hydrologists asked the author of the thesis for performing a trend analysis.

3.5 Trend analysis in hydrology in general

Although some advanced statistical hydrologists are of the opinion that the trend analysis is dead as stated, for instance, by the current president of the IAHS Commission on Statistical Hydrology (Grimaldi, 2014, personal communication), the methods devoted to it are still widely applied due to its stressed importance in hydrological modelling. However, indeed, some papers have recently emerged that question the concept of trend component in a time series itself because from various papers it is not clear what *trend* really is (or how it should be defined; see e.g. Fatichi et al., 2009). Currently, Sang et al. (2013) define trends as "the deterministic component in the analyzed data that corresponds to the biggest temporal scale on the condition of giving the concerned temporal scale". This definition does not exclude the cyclic component of very low (or the lowest discernible) frequency; so, in fact, the differences between cyclic (here, rather periodic than seasonal) and noncyclic component are somewhat blurred. Indeed, also the seasonal component can be considered trend from a certain point of view. On the other hand, some hydrological or climatological studies emphasize that, besides deterministic trends, there are also stochastic trends in geophysical time series, which have to be distinguished before modelling the future (Fatichi et al., 2009). Stochastic trends are closely connected to the issue of persistence that causes difficulties in finding deterministic trends.

Notwithstanding, in hydrology, trend is mostly thought of as noncyclic monotonic deterministic and, in the majority of cases, also linear component present in a time series. In this spirit (although the condition of linearity may often be omitted when using nonparametric techniques), the next chapters will be written as well. The study of currently emerging techniques would require undesirable delay and, therefore, here the author specialized in methods that were employed throughout papers A1–A9 predominantly.

3.5.1 Linear regression coefficient

Despite its well-known downsides, testing simple (i.e. bivariate) OLS linear regression coefficient (also referred to as slope) is still popular in hydrology. In fact, a t-test is applied to the coefficient β that, under the null hypothesis H_0, is Gaussian distributed with zero mean and variance σ_β^2, which can be formalized as $\beta \sim \mathcal{N}(0, \sigma_\beta^2)$. Recalling the model for the deseasonalized time series x_k, very similar to that in Eq. 3.9 except for the presence of the random error (white noise) component η_k, one may write (Machiwal and Jha, 2012)

$$x_k = \iota + \beta k + \eta_k \tag{3.30}$$

from which the concept of dependence of x_k on time k is apparent. Of course, it is assumed that also $\eta_k \sim \mathcal{N}(0, \sigma_\varepsilon^2)$. Another aspect where the model in Eq. 3.30 differ from that in Eq. 3.9 is that the coefficient ι (often called intercept) is nonzero ($\iota \neq 0$).

Bearing in mind what has just been said, the test statistic t_{OLS} is as follows

$$t_{OLS} = \frac{\hat{\beta}}{\sqrt{\sigma_{\hat{\beta}}^2}} \tag{3.31}$$

where the hats indicate estimates from the data. After some rearrangement of expressions in Machiwal and Jha (2012), the variance $\sigma_{\hat{\beta}}^2$ may be computed as

$$\sigma_{\hat{\beta}}^2 = \frac{\sum_{k=1}^{K} (x_k - \overline{x})^2 - \hat{\beta}^2 \sum_{k=1}^{K} (k - \overline{k})^2}{(K-2) \sum_{k=1}^{K} (k - \overline{k})^2} \tag{3.32}$$

with $\overline{k}$ indicating the average time index k and $\overline{x}$ the average of the series x_k. Then, the value of the statistic t_{OLS} can be compared with the quantiles of the Student t-distribution with $\nu = K - 2$ degrees of freedom. For instance, utilizing the two-sided version of the test, when $|t_{OLS}| > t_{1-\alpha/2,\nu}$ the regression coefficient is considered significant at the level of significance equal to α and the trend is found. If it is decreasing or increasing depends on the sign of $\hat{\beta}$ obtained by the normal equations.

There are several problems associated with this test. Namely, it does not distinguish among trend, persistence or periodicity. Problems with non-Gaussianity of the data are also reported (Hameed et al., 1997; Machiwal and Jha, 2012; Matalas and Sankarasubramanian, 2003; Yue et al., 2002b). Some hydrologist even do not check if the assumptions before applying this test are met (see e.g. Fanta et al., 2001).

3.5.2 Spearman's rho

Hydrologists often do not want to delay their analyses by checking the assumptions that parametric approaches require. Therefore, they rather employ nonparametric tests. One of these frequent applied tests is Spearman's rho (known also under several other names such as the Spearman rank order correlation (SROC) test; Machiwal and Jha, 2012). As the name suggests, it is just based on the Spearman correlation coefficient. The procedure may be described in several following steps

1. Assign ranks to data so that $RNK = 1$ belongs to the largest x_k and $RNK = K$ belongs to the smallest x_k. For the cases of ties, use the mean value of ranks.

2. Compute the differences D_k

$$D_k = RNK_k - k \tag{3.33}$$

3. Now, calculate the correlation coefficient resulting from the use of ranks

$$\rho = 1 - \frac{6\sum_{k=1}^{K} D_k^2}{K(K^2-1)} \tag{3.34}$$

4. Under the null hypothesis, $H_0 : \rho = 0$, the test statistic t_ρ computed as

$$t_\rho = \rho\sqrt{\frac{K-2}{1-\rho^2}} \tag{3.35}$$

has the Student t-distribution with $\nu = K - 2$ degrees of freedom. Thus, it can be compared with the quantiles $t_{1-\alpha/2,\nu}$ where α is the prescribed level of significance.

5. Hence, for the two-sided test that is usually employed, if $|t_\rho| > t_{1-\alpha/2,\nu}$, H_0 is rejected and a significant trend is found. The sign of ρ further specifies if the trend is increasing or decreasing.

By means of Monte Carlo simulations, Yue et al. (2002a) discovered that the SROC test has approximately the same power as the Mann–Kendall test (see Chapter 3.5.4). Since then, the Mann–Kendall test has become more popular in hydrology than the SROC test.

3.5.3 Cox–Stuart test for trend

In the years of a weak computers' capacity, a need for easy and quick tests arose. In the 1950s, Cox and Stuart (1955) proposed several simple tests among which a test for trends also appeared. Although this test is really old and has its drawbacks such as the sensitivity to persistence in data, some hydrologists or climatologists still use it and sometimes recommend it as being not sensitive to serial correlation, thus being more suitable for detecting deterministic trends than the Mann–Kendall test (see e.g. Sen and Niedzielski, 2010). These provocative suggestions led the author of the thesis to perform Monte Carlo simulations similar to those in Yue et al. (2002b), which resulted in valuable extended abstract that warns hydrologists against using the Cox–Stuart (CS) test when the series investigated is autocorrelated (Ledvinka, 2014b).

Nevertheless, the procedure of the CS test starts with splitting the series x_k into two halves $x_k^{(1)}$ and $x_k^{(2)}$, each of which has a length of K_0. If the number of observations K is an odd number, the $(K+1)/2$-th element is deleted. The test statistic is defined as follows (Sen and Niedzielski, 2010)

$$CS = \sum_{k=1}^{K_0} y_k \tag{3.36}$$

where

$$y_k = \begin{cases} 1 & \text{if } x_k^{(1)} < x_k^{(2)} \\ 0 & \text{otherwise} \end{cases} \tag{3.37}$$

Under the null hypothesis H_0, the statistic CS obeys a binomial distribution, that is $CS \sim Bi(L, p)$ with parameters $p = 0.5$ and L, which is the number of nonzero y_k. If $L < 20$ the decision on the rejection of H_0 must be made based on the exact distribution. On the contrary, the approximation by the standard Gaussian distribution may be used if the statistic CS is transformed to the statistic Z_{CS} using the estimates of its mean (subtracted term in the nominator) and variance (in the denominator) according to (Luo, 2013)

$$Z_{CS} = \frac{CS - L/2}{\sqrt{L}/2} \tag{3.38}$$

The values of Z_{CS} can then be compared with the quantiles of the standard Gaussian distribution. Therefore, if in the case of two-sided test $|Z_{CS}| > u_{1-\alpha/2}$ with α being the prescribed level of significance, the test rejects H_0 and finds the trend. The sign of the statistic Z_{CS} gives the information on the type of trend (i.e. prevailing drops or rises). Ledvinka (2014b), however, found that this test is sensitive to autocorrelation and its behaviour is even worse than that of the Mann–Kendall test; so it cannot be recommended for the trend analysis in hydrology.

3.5.4 Kendall's tau and the original Mann–Kendall test

The Mann–Kendall (MK) test is probably the most utilized trend test in current hydrology. Because it is in fact based on **the Kendall rank correlation coefficient also known as Kendall's τ (or tau)**, at this place, a short description of testing the null hypothesis H_0 that the distribution of data y_k does not change as a function of data x_k starts the description of the MK test. The procedure can, according to Hirsch et al. (1993), be summarized in several steps

1. Having K pairs of the data $(x_1, y_1), (x_2, y_2), \ldots, (x_K, y_K)$, sort them with respect to the magnitude of x_k so that $x_1 \leq x_2 \leq \ldots \leq x_K$.

2. Examine all $K(K-1)/2$ ordered pairs of y_k values. Let Q denotes the number of concordant cases where $y_\ell > y_j$, and M denotes the number of discordant cases where $y_\ell < y_j$, with indexes $\ell > j$.

3. Define the test statistic S_τ

$$S_\tau = Q - M \tag{3.39}$$

4. If $K \leq 10$, decide about the rejection of H_0 using the exact distribution of S_τ. Otherwise, as it is more frequent in literature, the approximation by the standard Gaussian distribution suffices. Since the mean of S_τ equals zero and the variance σ_τ^2, allowing the presence of ties, equals

$$\sigma_\tau^2 = \frac{K(K-1)(2K+5) - \sum_{g=1}^{K} t_g g(g-1)(2g+5)}{18} \tag{3.40}$$

the test statistic Z_τ is computed as follows

$$Z_\tau = \begin{cases} \dfrac{S_\tau - 1}{\sqrt{\sigma_\tau^2}} & \text{if } S_\tau > 0 \\ 0 & \text{if } S_\tau = 0 \\ \dfrac{S_\tau + 1}{\sqrt{\sigma_\tau^2}} & \text{if } S_\tau < 0 \end{cases} \tag{3.41}$$

Note that in Eq. 3.40, t_g denotes the number of ties with extent g among which also the ties representing unique values (i.e. one-element ties) are considered. At first glance, it may look strange but imagine for instance the dataset $5, 5, 6, 7, 8, 8, 8, 10, 10, 11, 12, 12$. Then, the t_g values are as follows: $t_1 = 3$ (three untied values $(6, 7, 11)$), $t_2 = 3$ (three ties of extent two $(5, 10, 12)$), $t_3 = 1$ (one tie of extent three (8)), and for all higher values of g, $t_g = 0$ (Hirsch et al., 1993).

5. The values of Z_τ may then be compared with the quantiles of the standard Gaussian distribution corresponding to some selected significance level α; so if $|Z_\tau| > u_{1-\alpha/2}$ and adopting the two-sided version of the test, H_0 is rejected and the influence of the variable x_k on the variable y_k is very likely significant.

6. To quantify the correlation coefficient τ bounded by the lower limit -1 and the upper limit 1, the statistic S_τ is divided by the total number of compared pairs of data. That is

$$\tau = \frac{S_\tau}{K(K-1)/2} \tag{3.42}$$

Using this coefficient, the relationships between Hurst exponents and p-values of trend tests were quantified in paper A4 and the relationships between standardized MK test statistics and mainly selected physiographic characteristics (such as coordinates of precipitation stations or centroids of hydrogeological regions) were examined in papers A6 and A5.

Bearing in mind that the time is only increasing variable, **the original MK test** inspecting the time series x_k can be derived very easily. First, the test statistic S_{MK} is defined as follows

$$S_{MK} = \sum_{\forall j < \ell} \operatorname{sgn}(x_\ell - x_j) \tag{3.43}$$

where $\operatorname{sgn}(\cdot)$ is the sign function and the indexes j and ℓ emphasize all possible pairs composed of earlier and later values of a deseasonalized (e.g. annual) series x_k. In fact, the rest of the testing procedure is the same as in the case of Kendall's tau. Often, the exact distribution of the statistic S_{MK} is approximated by the standard Gaussian distribution when there are enough observations (say $K > 10$; Kendall, 1970). Again, the quantification of its variance σ_{MK}^2 is necessary and then the standardized test statistic is compared with the quantiles of the standard Gaussian distribution (see papers A4, A5, A6 and A9). In its original form, the MK test was applied in papers A1 and A2.

The MK test statistic S_{MK} is usually accompanied by a nonparametric estimator of the linear slope of the trend, **the so-called Sen's slope estimator (or the Theil–Sen slope estimator)**. This is done for practical purposes when one needs to quantify the trend in physical units such as mm in the case of precipitation. Among other reasons belongs the fact that this estimator is used in some modification of the MK test proposed (see e.g. Chapter 3.7).

The Sen slope estimator (Sen, 1968) is actually the median value of all the slopes given by all the possible pairs among data forming the time series x_k. Again, using indexes j and ℓ in a similar way as in Eq. 3.43, it can be formally written as

$$\beta_S = \text{med}\left(\frac{x_\ell - x_j}{\ell - j}\right), \quad \forall j < \ell \tag{3.44}$$

As in the case of simple OLS linear regression, also the intercept parameter ι_S may be added to this type of slope to allow the construction of trend lines in plots (see papers A7 and A9). It is computed as (e.g. Bronaugh and Werner, 2013)

$$\iota_S = \overline{x} - \beta_S \overline{k} \tag{3.45}$$

with k and horizontal bars indicating time and averages, respectively.

3.6 The issue of persistence in trend analysis

Unfortunately, all the above trend tests, without exception, are sensitive to serial correlation, which in fact means the violation of independence among data. Indeed, one of the most frequent assumptions before the application of these tests is just the independence, which holds true also for the tests devoted to the detection of abrupt changes as documented by Yue and Wang (2002a). Even the MK test is affected by autocorrelation, which was confirmed by Monte Carlo simulations performed in Kulkarni and von Storch (1995); Yue et al. (2002b).

From previous studies, it is apparent that the violation of independence somewhat alternate the variances of test statistics. While the positive serial correlation inflates their distribution, the negative serial correlation, which is not so typical in hydrology but does exist, deflates it (see e.g. Rivard and Vigneault, 2009). Of course, this alteration of variances is closely tied with the type I and type II errors of the tests (Önöz and Bayazit, 2012). Therefore, when not properly considered in the computation of variances, the positive serial correlation causes the tests to detect trends too often, whereas the negative serial correlation has the opposite effect. Thus, a lot of modifications of the trend tests came into being tackling the issue of the presence of autocorrelation either by working directly with the expressions for variances or utilizing resampling techniques such as different types of bootstrap designed for time series. A lot of these modifications were developed by hydrologists, many of them being linked to the MK test in particular, and it is really difficult to orientate in literature where, sometimes, it is hard to distinguish which of the modifications was exactly applied and why. Note that, hereinafter, the term short-term persistence (STP) will be frequently used in an attempt to distinguish between these effects from those induced by long-term persistence briefly described in the next paragraph.

In recent years, a need for addressing long-term persistence (LTP) in hydrology and climatology has been emphasized by some researchers because from mathematical theory it is evident that it can, similar to the effects of STP mentioned in the preceding paragraph, stand behind falsely detected deterministic trends whose certain proportion should rather be considered stochastic (e.g. Cohn and Lins, 2005; Fatichi et al., 2009). However, not only trends but also cyclic components such as periodicity of a series may be triggered by LTP (Cohn and Lins, 2005). Note that only some aspects regarding LTP were addressed by the author, others were rather deferred to his future work. Note also that LTP appears in literature under various other names such as long range dependence and is closely related to stochastic models accounting for long memory in time series.

3.6.1 Short-term persistence

The effect of STP on the results of the MK test was known to the author from the very beginning of his Ph.D. study. Therefore, he focused primarily on modifications accounting for it. Typical time series model (or stochastic model) capable of describing and simulating the behaviour of a time series in general is a linear process, truncating of which led to the establishment of the practical Box–Jenkins methodology that makes the correlation structure of a time series beneficial for forecasting purposes (Box et al., 2008). Exactly this was the reason why the role of correlated stochastic component ε_t of a general time series x_t was stressed in Eq. 1.1.

The general Box–Jenkins model that mimic STP (because their autocorrelation function, ACF, usually quickly approaches zero) is a stationary (mixed) autoregressive-moving average (ARMA) model. This model is frequently written in terms of its autoregressive order p and moving average order q characterizing how many past observations are considered when building the model. Depending on the orders, the number of its autoregressive parameters ϕ_p, moving average parameters ϑ_q plus the other two parameters μ_x (level of the process) and σ_ε^2 (variance of the white noise process ε_t) have to be estimated. Considering a general time series x_t that is (deterministically) detrended, deseasonalized and its level equals zero, ARMA models may be written as

$$x_t - \phi_1 x_{t-1} - \ldots - \phi_p x_{t-p} = \varepsilon_t - \vartheta_1 \varepsilon_1 - \ldots \vartheta_q \varepsilon_{t-q} \tag{3.46}$$

which can be (using usual autoregressive and moving average polynomials $\Phi(B)$ and $\Theta(B)$) briefly written as (e.g. Box et al., 2008; Brockwell and Davis, 1991)

$$\Phi(B)x_t = \Theta(B)\varepsilon_t \tag{3.47}$$

where B is the backshift operator having such an effect that $Bx_t = x_{t-1}$.

In hydrological practice, however, mainly a part of ARMA(p, q) models known as the lag-one (first-order) autoregressive process AR(1) has been relevant due to its fitting abilities to observed time series such as (annual) discharge. This is also the reason why the modifications of trend tests take into account this process particularly in an attempt to suppress its influence on variances of test statistics when examining time series for the presence of deterministic trends. The AR(1) process is often written in an inverted form using the lag-one autocorrelation coefficient r_1 because, in this particular case, the only autoregressive parameter $\phi_1 = r_1$. That is

$$x_t = r_1 x_{t-1} + \eta_t \tag{3.48}$$

where η_t is usually considered *iid* and $\eta_t \sim \mathcal{N}(0, \sigma_\eta^2)$, similar to the elements of the vector $\boldsymbol{\eta}$ in Eq. 3.3. Note that for the stationary models AR(1), it is required that $|r_1| < 1$. The next chapter deals with the estimation of r_1 from observed data.

3.6.2 Lag-one autocorrelation coefficient and its significance

Although many other estimators of the lag-one autocorrelation coefficient r_1 were proposed in the past (see e.g. Solanas et al., 2010; Yevjevich, 1972), in hydrology the following one, in fact resulting from the PMCC, is predominantly used (Salas et al., 1980)

$$\hat{r}_1 = \frac{\sum_{t=1}^{T-1} x_t x_{t+1}}{\sum_{t=1}^{T} x_t^2} \tag{3.49}$$

Note that in 3.49 is still assumed that the series level equals zero. Otherwise, the average of the series x_t should be subtracted from it before estimating r_1. Note also that in the denominator on the right hand side of Eq. 3.49 it is assumed the the standard deviations of series x_t and x_{t+1} do not differ significantly and thus the variance (or sum of squares after excluding the multiplication factor) of the series x_t is used there.

When deciding on the significance of r_1, in hydrology, the so called Anderson test is applied, keeping in mind that the estimator $\hat{r}_1$ is biased downwards. To reject the null hypothesis $H_0 : r_1 = 0$ (i.e. there is no persistence in x_t), the estimate must fall outside the region given by the limits (Anderson, 1942; Machiwal and Jha, 2012; Salas et al., 1980)

$$\hat{r}_1 = \frac{-1 \pm u_{1-\alpha/2}\sqrt{T-2}}{T-2} \tag{3.50}$$

where $u_{1-\alpha/2}$ is the quantile of the standard Gaussian distribution corresponding to the prescribed level of significance α.

Testing the significance of r_1 this way was performed in papers A4 and A6 where a new code for the BHMLLESS–MK test (see Chapter 3.7.2) had to be created for `R`, while this software uses different decision rule (see Cowpertwait and Metcalfe, 2009).

3.6.3 Long-term persistence

Sometimes, it may happen that the time series (and then the process that generates it) turns out to be (stochastically) nonstationary which means that the autoregressive polynomial $\Phi(B)$ has some roots within or on the unit circle in the complex plane. For these cases a generalization of ARMA processes was suggested in TSA in which the series is differenced once or twice prior to the application of ARMA processes themselves in order to stationarize the original nostationary series (e.g. Box et al., 2008). These models are called autoregressive integrated moving average (ARIMA) processes where one additional, so-called differencing, parameter d appears in their representation. The whole brief representation of ARIMA(p, d, q) then looks as follows

$$\Phi(B)(1-B)^d x_t = \Theta(B)\varepsilon_t \tag{3.51}$$

where the remaining parameters are the same as in Eq. 3.47.

In Eq. 3.51, the parameter d is assumed to be integer exclusively. Yet, there are other processes that are more important in hydrology than ARIMA processes and that incorporate the parameter d relaxed to take on fractional values (usually $d \in (-0.5, 0.5)$ or $d \in (0, 0.5)$). These are called fractionally integrated autoregressive-moving average (FARIMA or ARFIMA) processes whose relationship to older fractional Gaussian noises (FGN), capable of modelling and simulating natural and hydrological time series revealing long memory, was studied in Hosking (1981) and his later works (e.g. Hosking, 1984). Exactly the fact that FARIMA models are able to deal with long memory time series makes them very popular in recent hydrology (Montanari, 2003; Montanari et al., 1997). Their ACF decays hyperbolically rather than exponentially which is a manifestation of LTP.

In hydrological applications, mainly FARIMA$(0, d, 0)$ was used, which means that the fractional differencing parameter d suffices to be estimated because there are no autoregressive nor moving average parts necessary according to some experience (see e.g. Khaliq et al., 2008). It may be shown that the parameter d can be expressed in terms of the Hurst exponent H. That is (e.g. Khaliq and Sushama, 2012)

$$H = d + 0.5 \tag{3.52}$$

where the Hurst exponent H may be defined in terms of the asymptotic behaviour of the rescaled range $RR(N)$ as a function of the time span N of a time series as (e.g. Feder, 1988)

$$\mathrm{E}\left[\frac{RR(N)}{SS(N)}\right] = CN^H \quad \text{as } N \to \infty \tag{3.53}$$

where $SS(N)$ is the standard deviation of the first N values incorporated in the range, $\mathrm{E}[\cdot]$ means the expected value and C is a constant.

Thus, it is of great importance to study techniques through which one may estimate this exponent (see Chapters 3.6.5 and 3.6.6). Sometimes, it is questionable if the time series be modelled by an ARMA or a FARIMA model. In modern TSA, the so-called unit root tests may help with this decision when addressing this question (see Chapter 3.6.4 which follows immediately).

3.6.4 Unit root testing

Unit root testing was virtually borrowed from econometrics where, in fact, a lot of tests devoted to investigating if the time series should be characterized by a process (especially ARIMA) whose autoregressive polynomial has a root (or more roots) very close to unity (e.g. Banerjee et al., 1993; Dickey and Fuller, 1979; Fuller, 1996; MacKinnon, 1996; Phillips and Perron, 1988). In hydrology and climatology, the (augmented) Dickey–Fuller (DF) test and the Phillips–Perron (PP) test have been applied (see e.g. Barbosa et al., 2008; Fatichi et al., 2009; Jarvis et al., 2013; Modarres and Ouarda, 2013; Rutkowska and Ptak, 2012; Wang et al., 2006). These test are often combined with the so-called Kwiatkowski–Phillips–Schmidt–Shin (KPSS) stationarity test of (Kwiatkowski et al., 1992). Although sometimes considered a unit root as well (as done in papers A3, A5 and A6 for the sake of brevity), this test has in fact its null hypothesis (or hypotheses) set in an opposite way to the unit root tests. This combination allows to distinguish more types of stochastic processes that to discriminate only between unit root process and (stochastically) stationary processes. Moreover, some combinations allow for the discrimination between deterministic and stochastic nonstationarities (trends).

Inspired by the works of Barbosa (2011); Barbosa et al. (2008) or Fatichi et al. (2009), the author of the thesis applied a combination of the PP and KPSS tests to 7-day low flows observed at 144 Czech water-gauging stations (paper A3), to technical precipitation series from 268 Czech meteorological stations (paper A6) and several spring-yield series from Czechia (i.e. only those containing no missing values; paper A5). Without knowing that there are some more sophisticated `R` packages (see e.g. McLeod et al., 2012; Pfaff, 2008; Zhang et al., 2013), the author used exclusively the `tseries` package (Trapletti and Hornik, 2016) which incorporates the functions `pp.test()` and `kpss.test()`.

Throughout papers A3, A5 and A6, the PP test (Fatichi et al., 2009; Phillips and Perron, 1988) was based on the model

$$x_t = \iota + \beta \cdot t + \phi x_{t-1} + \varepsilon_t \tag{3.54}$$

where ι and β were the parameters of a linear regression and ε_t was the stationary process allowed to be serially correlated or heteroscedastic. When the null hypothesis of a random walk, i.e. $H_0 : \phi = 1$, was rejected at some level of significance α (critical values are specific and are tabulated in literature or software) in favour of the alternative hypothesis, $H_1 : \phi < 1$, the stationary AR(1) process combined with a deterministic linear trend was found as underlying the time series. Note that only one-sided version of the PP test is used because the situation with coefficients $\phi \leq -1$ is not so important in practice (see e.g. Cipra, 2008).

The KPPS test (Fatichi et al., 2009; Kwiatkowski et al., 1992), on the contrary, was based on the model

$$x_t = \beta \cdot t + \varrho_t + \nu_t \tag{3.55}$$

where, again, β was the slope of a linear regression, ϱ_t a random walk such that

$$\varrho_t = \varrho_{t-1} + \eta_t, \quad \eta_t \sim \mathcal{N}(0, \sigma_\eta^2) \tag{3.56}$$

and ν_t was a stationary process. One may select between two null hypotheses associated with this test, namely $H_0^{\mathrm{a}} : \sigma_\eta^2 = 0, \beta = 0$ or $H_0^{\mathrm{b}} : \sigma_\eta^2 = 0, \beta \neq 0$ against the alternative hypothesis $H_1 : \sigma_\eta^2 > 0$. Exactly H_0^{b} was necessary since the purpose of the analyses in papers A3, A5 and A6 was to distinguish between deterministic trend and stochastic nonstationarity.

As can be deduced, according to the results of the two tests, up to four situations may appear then

1. stationary short memory processes about a deterministic trend if the null hypothesis of the PP test is rejected and the null hypothesis of the KPSS test is accepted;

2. unit root processes along with random walks if the null hypothesis of the PP test is accepted and the null hypothesis of the KPSS test is rejected;
3. other (stochastically) nonstationary process (including difference stationary long memory processes) if both null hypotheses are rejected;
4. failure of the tests resulting from insufficient information in data.

3.6.5 Maximum-likelihood estimators of the Hurst exponent

In papers A3 and A6, the unit root tests were also accompanied by the estimates of the Hurst exponent, or keeping in mind Eq. 3.52, the fractional differencing parameter d. Leaving aside the Bayesian approach (see e.g. Makarava, 2012), the maximum likelihood (ML) estimators are considered best from different viewpoints, some of them having been developed directly in hydrology and adapted in other disciplines (e.g. Navarro et al., 2013). Therefore, in papers A3 and A6 exactly the ML estimation was utilized. However, in fact, the methods used were not identical because in the preparation of paper A3, the second ML estimator had not been implemented in any of the `R` packages. In particular, the `forecast R` package (Hyndman and Khandakar, 2008) and very recently developed `HKprocess R` package (Tyralis, 2016) were employed.

The first package actually uses the `fracdiff R` package (Fraley et al., 2012) that implements the approach of Haslett and Raftery (1989) when estimating the fractional differencing parameter d. Referring to Hosking (1981), they developed a good approximation which is not as computationally demanding as other ML estimators. The procedure may be described in several following steps

1. First, it is necessary to note that a general time series x_t, conditionally on x_{t-1}, has a Gaussian distribution with expected value $\mathrm{E}(x_t|x_{t-1})$ and variance $\mathrm{var}(x_t|x_{t-1})$, respectively. While Hosking (1981) calculated them based on the autocorrelations of full FARIMA(p, d, q) process using the Drubin–Levinson recursion (e.g. Brockwell and Davis, 1991), Haslett and Raftery (1989) found that working with the autocorrelations of FARIMA$(0, d, 0)$ is sufficient for those estimates.
2. The estimate of the expected value is then expressed as

$$E(x_t|x_{t-1}) = u_t + w_t\mu \tag{3.57}$$

and the estimate of the variance as

$$\mathrm{var}(x_t|x_{t-1}) = \upsilon_t = \sigma_x^2\kappa\prod_{j=1}^{t-1}(1-\phi_{jj}^2) \tag{3.58}$$

with

$$u_t = \Phi(B)\Theta(B)^{-1}\sum_{j=1}^{t-1}\phi_{tj}x_{t-j} \tag{3.59}$$

and

$$w_t = 1-\Phi(1)\Theta(1)^{-1}\sum_{j=1}^{t-1}\phi_{tj} \tag{3.60}$$

where ϕ_{tj} are partial linear regression coefficients of FARIMA$(0, d, 0)$ (for details see Haslett and Raftery, 1989) and κ is the ratio of the innovations variance to the process variance of ARMA(p, q) with parameters $\Phi(B)$ and $\Theta(B)$.

3. Having values of d, $\Phi(B)$ and $\Theta(B)$, the approximate estimates of μ and σ_η^2 may be respectively acquired as

$$\hat{\mu} = \frac{\sum_{t=1}^{T} w_t (x_t - u_t) \upsilon_t^{-1/2}}{\sum_{t=1}^{T} w_t^2} \tag{3.61}$$

and

$$\hat{\sigma}_\eta^2 = \frac{1}{T} \sum_{t=1}^{T} \frac{(x_t - u_t - w_t \hat{\mu})^2}{\upsilon_t} \tag{3.62}$$

with u_t, w_t and υ_t defined in step (2).

4. Given these estimates, the approximate log-likelihood may then be written as

$$l(d, \Phi(B), \Theta(B)) = -\frac{T}{2}(\log \hat{\sigma}_\eta^2 + 1) + \text{const} \tag{3.63}$$

which should be maximized with regard to different values of parameters. However, because in papers A3, only the parameter d was required for the FARIMA$(0, d, 0)$ process, the numerical maximization was quicker.

The second package makes use of the findings of Tyralis and Koutsoyiannis (2011) who built on McLeod and Hipel (1978) and McLeod et al. (2007). Their so-called Hurst–Kolmogorov process (HKp), that has its close relationships to FGN and FARIMA$(0, d, 0)$ processes given by different philosophical meaning (see e.g. Koutsoyiannis, 2010), is characterized by a vector of parameters $\boldsymbol{\Xi} = (\mu_x, \sigma_x, H)$ where μ_x and σ_x stand for the mean and standard deviation of the process, while H is just the Hurst parameter which had to be found in paper A6. Suppose for now also that the observations x_t are stored in a vector $\mathbf{x}_T = (x_1, x_2, ..., x_T)^\top$. The general form of the likelihood is

$$L(\boldsymbol{\Xi}|\mathbf{x}_T) = \frac{1}{\sqrt{(2\pi)^T \det(\sigma_x^2 \mathbf{A})}} \exp\left[-\frac{(\mathbf{x}_T - \mu_x \mathbf{e})^\top \mathbf{A}^{-1} (\mathbf{x}_T - \mu_x \mathbf{e})}{2\sigma_x^2}\right] \tag{3.64}$$

where $\mathbf{e}$ is a column vector containing T unities, $\mathbf{A}$ is the T by T autocorrelation matrix and $\det(\cdot)$ denotes the determinant of the matrix. The estimate of H can be obtained by minimizing the single-variable function $G(H)$ given by the following Eq. 3.65

$$G(H) = -\frac{T}{2} \ln \left[\left(\mathbf{x}_T - \frac{\mathbf{x}_T^\top \mathbf{A}^{-1} \mathbf{e}}{\mathbf{e}^\top \mathbf{A}^{-1} \mathbf{e}} \mathbf{e} \right)^\top \mathbf{A}^{-1} \left(\mathbf{x}_T - \frac{\mathbf{x}_T^\top \mathbf{A}^{-1} \mathbf{e}}{\mathbf{e}^\top \mathbf{A}^{-1} \mathbf{e}} \mathbf{e} \right) \right] - \frac{1}{2} \ln\left[\det(\mathbf{A})\right] \tag{3.65}$$

Note that there is a possibility to test the significance of the Hurst exponent. Uncorrelated white noise process should have this parameter very close to the value of 0.5, while persistence is characterized by $H \in (0.5, 1)$ and antipersistence by $H \in (0, 0.5)$ (see Khaliq et al., 2008; Khaliq and Sushama, 2012). Although in the past, there were developed several analytical techniques (e.g. Couillard and Davison, 2005; McLeod and Hipel, 1978), hydrologists rather employ Monte Carlo simulations where a large number of samples of white noise is generated and then some estimator is applied. When the original time series reveals values of H falling outside the confidence interval given by the empirical quantiles corresponding to the type I error, H is considered significant (Khaliq et al., 2009a, 2008). Of course, every estimation method is typical of its own uncertainty, especially as regards small samples in hydrology (Tyralis and Koutsoyiannis, 2011) and it should definitely be accounted for in the process of testing for the significance of H. The author of the thesis attempted to develop such a method based on Monte Carlo simulations and the maximum entropy bootstrap (MEB; Vinod, 2006; Vinod and López-de Lacalle, 2009; Srivastav and Simonovic, 2014) through which the uncertainty in the estimation may be quantified (Barbosa, 2011; Barbosa et al., 2011; Monteiro et al., 2012), but currently the author's code is really computationally demanding and cannot be put in practice, particularly in conjunction with daily data as stated in paper A6. Therefore, in papers A3 and A6, no significance testing was performed, despite it would improve the capabilities of the trend test proposed in 3.8.3.

3.6.6 Other estimators of the Hurst exponent

As already mentioned, there are not only the estimators of the Hurst exponent based on the maximum likelihood techniques. Traditionally, so-called heuristic or semiparametric techniqes such as the rescaled range statistic (RRS) of Hurst (1951) has been used to which a lot of additional techniques were suggested (e.g. Fatichi et al., 2009; Taqqu et al., 1995). The are also methods whose basis lies in the frequency domain (e.g. Geweke and Porter-Hudak, 1983). Basically, each of these methods works with double logarithmic spectra where some selected statistic (e.g. variance) is plotted against varying scale of time series. The spectra often reveal that the magnitude of the statistic change throughout different scales. In double logarithmic plots, a trend (often linear) can be fitted to the points whose slope then essentially determines the estimate of the Hurst exponent H.

Recently, also wavelet techniques were designed for this purpose. Although it was not addressed in the attached papers, it should be emphasized that Ledvinka (2014a) through a wavelet-based method outlined in Fatichi et al. (2009) found that in the Ore Mountains, Czechia, the Hurst phenomenon may be present in selected small catchments regarding discharge series which may be illustrated by Fig. 3.2. This figure was made with the help of the `R` package `wmtsa` (Constantine and Percival, 2013) to which an additional code was written so that it was possible to fit linear trends using weighted least squares (WLS; Barbosa et al., 2008; Percival and Walden, 2000). This finding regarding the Ore Mountains is somewhat contradictory compared to studies such as Mudelsee (2007) on one hand, but may also explain the patterns mentioned in Szolgayova et al. (2014a,b). For further details, see Chapter 5.2 devoted to discussion.

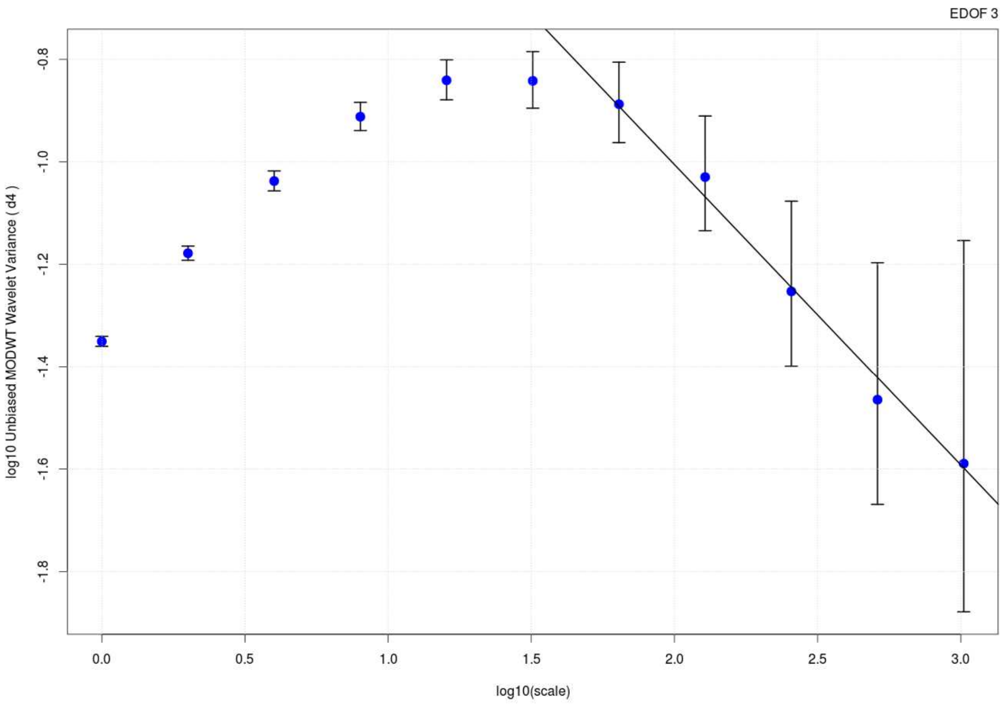

Figure 3.2: An example of double logarithmic plot depicting the dependence of the maximal overlap discrete wavelet transform variance on the scale (case of the discharge series observed at Rothenthal on the Natschung Brook in the Ore Mountains; adapted from Ledvinka, 2014a)

3.7 Modifications of trend tests

To overcome the issues caused by persistence in their time series, hydrologists developed several modification of trend tests. The majority of them was based od the MK test, a few of which were utilized by the author of the thesis as well. In the following list, tests relevant to attached paper and some more are briefly described to show how they work. Mostly those dedicated to STP were explored by the author and sometimes his own `R` scripts had to be written because there was no package incorporating the tests.

3.7.1 Trend-free pre-whitening (TFPW) MK test

Probably this is the modification of the MK test which is best known because of its implementation in the `zyp` `R` package (Bronaugh and Werner, 2013). It is based on so-called pre-whitening which has been known for more than 20 years but the paper of Kulkarni and von Storch (1995) was strongly influential. In 2002, Yue et al. (2002b) adopted the concept of pre-whitening and having been influenced further by the paper of Hamed and Rao (1998) realized that there is an interaction between deterministic trend and persistence (namely STP) in a hydrological series. Therefore, their procedure starts with detrending (i.e. subtracting the term $\beta_S k$ from an annual series x_k) the series using the Sen slope estimator in Eq. 3.44 whose confidence limits may be computed using Wagesho et al. (2012)

$$M_{1,2} = \frac{K(K-1)/2 \pm u_{1-\alpha/2} \cdot \sqrt{\sigma^2_{MK}}}{2} \tag{3.66}$$

where $u_{1-\alpha/2}$ represents the $1-\alpha/2$ quantile of the standard Gaussian distribution and σ^2_{MK} the variance of the original MK test statistic S_{MK}. Then, the pre-whitening approach in terms of subtracting the AR(1) process, ideally only if the lag-one autocorrelation coefficient r_1 (Eq. 3.49) seems to be significant (Eq. 3.50) as follows

$$\eta_k = x^D_k - r_1 x^D_{k-1} \tag{3.67}$$

where x^D_k denotes the detrended series. Then, the trend is added back to the series and a new, pre-whitened series y_k of length $K-1$ is created. That is

$$y_k = \eta_k + \beta_S k \tag{3.68}$$

After the pre-whitening process, the original MK test from Chapter 3.5.4 may be applied to the series y_k. Using this test, at-site trends were detected in papers A4, A8 and A9. Also, it was a basis for the detection of field significant trends in paper A5 (Chapter 3.8.1)

3.7.2 Equivalent sample size modifications of the MK test based on data

Another approach how to deal with persistence in a time series during the detection of trends is multiplying the variance of the test statistic by some factor, the quantifying of which is not a trivial task (see e.g. Thiébaux and Zwiers, 1984). Mainly STP was addressed in the past this way.

In fact, two versions of the MK test of this type came into being. Lettenmaier (1976) based his findings on Bayley and Hammersley (1946) and Matalas and Langbein (1962). Therefore, although he originally worked with testing the SROC (Chapter 3.5.2), during the time, the modification of the MK test called the Bayley–Hammersley–Matalas–Langbein–Lettenmaier equivalent sample size (BHMLLESS) MK test was developed that made use of the same factor multiplying the variance of the original MK test statistic σ^2_{MK}. If the lag-one serial correlation r_1 proves to be significant (see Eq. 3.50), then the modified variance of the MK statistic $\tilde{\sigma}^2_{MK}$ is computed as follows

$$\tilde{\sigma}^2_{MK} = \sigma^2_{MK} \cdot \left[1 + 2 \cdot \frac{r_1^{K+1} - K \cdot r_1^2 + (K-1) \cdot r_1}{K \cdot (r_1 - 1)^2}\right] \tag{3.69}$$

While in the classical BHMLLESS–MK test the lag-one serial correlation coefficient is computed directly, the second version (Yue and Wang, 2004) requires the linear trend expressed as Sen's slope to be subtracted from the investigated series first. Again, the significance of the slope may be tested prior to its subtracting (Eq. 3.66). Exactly, this second version was used in both papers A4 and A6, although, due to space limitation, it was abbreviated as the YW–MK test (i.e. the Yue–Wang MK test). The author had to write his own `R` code for this purpose which is prepared for other hydrologists as well.

3.7.3 Equivalent sample size modification of the MK test based on ranks of data

Adopting the results of Bayley and Hammersley (1946), Hamed and Rao (1998) pointed out that rather the autocorrelation of ranks instead of data themselves should be accounted for when estimating the equivalent sample size for computing the variance of the MK test. Thus, they emphasized that the variance of the original MK test statistic σ^2_{MK} should be multiplied by a factor that takes into account the loss of the information contained in the autocorrelated data according to

$$\tilde{\sigma}^2_{MK} = \sigma^2_{MK} \cdot \frac{K}{\tilde{K}} \tag{3.70}$$

which was, after some empirical experiments, particularly expressed as (Hamed and Rao, 1998)

$$\tilde{\sigma}^2_{MK} = \sigma^2_{MK} \cdot \left[1 + \frac{2 \cdot \sum_{h=1}^{K-1} (K-h)(K-h-1)(K-h-2) r_h^{RNK}}{K(K-1)(K-2)} \right] \tag{3.71}$$

with r_h^{RNK} being the lag-h autocorrelation coefficient obtained from ranks of data after subtracting the Sen nonparametric estimate of linear trend from the data (3.44).

Notice that this modification was thought of as general, not merely accounting for the lag-one autocorrelation, and in this meaning Khaliq et al. (2009a) explored its abilities in comparison to a general form of the MK test from Chapter 3.7.2 taking account of the autocorrelation at larger lags as well. Nevertheless, to the author's knowledge, these experiments have not been carried out in Czechia with hydrometeorological data.

This test was not used in any of the papers attached. However, a function from the `R` package `fume` (Santander Meteorology Group, 2012) involving this type of the MK test was shortly tested without reasonable results relative to the territory of Czechia. It would definitely require more thorough inspection, but the package `fume` is now archived.

3.7.4 Equivalent sample size modification of the MK test accounting for long-term persistence

With rising interest in LTP, also a modification of the MK test incorporating the correction of the variance/standard deviation of its statistic was developed in this manner. When dealing with trends in the timing of drought spells in Canada, Ehsanzadeh and Adamowski (2010), made use of the work of Koutsoyiannis (2003) and Beran (1994) and applied his equivalent sample size approach where the Hurst exponent H appeared. The modified variance of the MK statistic under the LTP hypothesis can then be written as

$$\tilde{\sigma}^2_{MK} = \sigma^2_{MK} \cdot \frac{K - 1/2}{K - K^{2H-1}} \tag{3.72}$$

This can be wisely coupled with the MK test modification dealing with STP, which can result in a GESS–MK test 3.8.3.

3.7.5 Hamed's modification of the MK test accounting for long-term persistence

Ten years later after proposing their STP modification of the MK test, the first author of Hamed and Rao (1998), came up with another modification of the MK test, this time accounting for LTP or, in other words, scaling (Hamed, 2008).

The procedure of this tests may be summarized in a few following steps

1. Perform the original MK test starting with Eq. 3.43. If the trend turns out to be insignificant, the procedure may stop. Otherwise, proceed to step 2.

2. Subtract the Sen nonparametric trend (Eq. 3.44) from the data. From detrended data, compute the equivalent Gaussin variates z_k according to

$$z_k = \Phi^{-1}\left(\frac{RNK_k}{K+1}\right) \tag{3.73}$$

   where Φ^{-1} is the inverse standard Gaussian distribution function and RNK_k represent the rank of detrended (annual) observations x_k^D.

3. Using these variates stored in a vector $\mathbf{z}$ and an autocorrelation matrix for a given Hurst exponent $\mathbf{A}_H$, maximize the following log-likelihood

$$\log L(H) = -\frac{1}{2} - \det(\mathbf{A}_H) - \frac{\mathbf{z}^\top \mathbf{A}_H^{-1}\mathbf{z}}{2\sigma_z^2} \tag{3.74}$$

   where σ_z^2 is the variance of variates z_k.

4. Decide about the significance of H which is approximately Gaussian distributed. For this purpose Hamed (2008) derived empirical relationships for the following mean and standard deviation of H

$$\mu_H = 0.5 - 2.874K^{-0.9067} \tag{3.75}$$

   and

$$\sigma_H = 0.77654\sqrt{K} - 0.0062 \tag{3.76}$$

   If H proves to be close to the value of 0.5, the trend may be considered significant. Otherwise, a modified version of the MK test must be applied which is described in the following steps.

5. Compute the modified variance of the MK test statistic $\tilde{\sigma}^2_{MK}$ according to

$$\tilde{\sigma}^2_{MK} = \sum_{i<j}\sum_{k<l} \frac{2}{\pi}\sin^{-1}\left(\frac{r_{jl} - r_{il} - r_{jk} + r_{ik}}{\sqrt{(2-2r_{ij})(2-2r_{kl})}}\right) \tag{3.77}$$

   with indexes i, j, k and l satisfying the inequalities under the sums so that the correlations between the standard Gaussian variates $r_{..}$ indicated by the same indexes could be quantified.

6. Correct the variance $\tilde{\sigma}^2_{MK}$ for the bias by multiplying it by the factor BI according to (Hamed, 2008)

$$BI = a_0 + a_1 H + a_2 H^2 + a_3 H^3 + a_3 H^4 \tag{3.78}$$

with regression coefficients quantified empirically as

$$a_0 = \frac{1.0024K - 2.5681}{K + 18.6693} \tag{3.79}$$

$$a_1 = \frac{-2.2510K + 157.2075}{K + 9.2245} \tag{3.80}$$

$$a_2 = \frac{15.3402K - 188.6140}{K + 5.8917} \tag{3.81}$$

$$a_3 = \frac{-31.4258K + 549.8599}{K - 1.1040} \tag{3.82}$$

and

$$a_4 = \frac{20.7988K - 419.0402}{K - 1.9248} \tag{3.83}$$

7. Putting the modified variance $\tilde{\sigma}^2_{MK}$ into the expression, where the standardized test statistic is computed similar to that in Eq. 3.41, one may decide whether the trend is still significant or not. If so, a significant deterministic trend is found regardless of the presence of LTP in the investigated time series.

Very recently, this test was incorporated in the `R` package `HKprocess` (Tyralis, 2016), but it has not been explored carefully as regards Czech hydrometeorological data. Despite it is reported that it has some problems with ties Hamed (2008), it undoubtedly deserves its attention. Note also, the the author of this test dedicated another paper to detecting trends when he derived the exact distribution of the MK test for small sample sizes revealing persistence and concluded that the Beta distribution approximates better such data than the Gaussian distribution (Hamed, 2009).

3.7.6 Other valuable tests based on the MK test

There are also other tests built on the MK test that are worth as regards papers A1, A2 and A6. First, it is important to note the the seasonal version of the MK test really does exist that was developed by Hirsch and Slack (1984) and later mentioned, for instance, by Machiwal and Jha (2012). However, although in papers A1 and A2, there are some references to a seasonal MK test, in fact the seasonal MK test statistic was not considered there. Rather, we quantified only the statistics pertaining to each season (i.e. month) independently. The reason was that throughout the year, trends with different signs can be observed in Czechia which is contradictory to the assumption of consistency mentioned, for example, by Salas (1993). The fact that papers A1 and A2 mention this modification was rather an artefact of at-that-time using MS Excel macro related to the paper Libiseller and Grimvall (2002).

During the preparation of paper A6, several figures were produced that depict the sequential development of the standardized MK statistic of the BHMLLESS modification that made use of the 30-year moving window, which was an approach very similar to that in Hannaford and Buys (2012). The rationale for using such windows was that it was necessary to incorporate also the sequential quantification of the lag-one autocorrelation coefficients that would be more biased if shorter windows were used. Through the utilization of his own scripts, the author examined every single variable appearing in this paper, but, due to space limitation in the manuscript, only a plot depicting the situation regarding an annual precipitation produced by averaging over all 268 meteorological stations could be shown there. On the other hand, this approach cannot be compared to what is traditionally called the sequential MK test since the retrograde analysis was not conducted as, for instance, in Sneyers (1990) or Tabari et al. (2015).

Probably, there are more valuable trend test based on or coupling the MK test or its modifications that are unknown to the author, or the author knows only a very little bit of these more advanced techniques. The wavelet-based MK tests may serve as a good example (see e.g. Adamowski et al., 2009; Partal, 2012, 2010; Partal and Küçük, 2006).

3.7.7 Trend tests and (block) bootstrapping

Bootstrapping that builds on the jackknife approach (see e.g. Davison and Hinkley, 1997; Efron and Tibshirani, 1993) is a powerful resampling technique that can be appropriately coupled with trend tests where the function giving the test statistic may be bootstrapped. In general, the methods developing this way deserve its attention because they have a great potential in hydrology where analytical solution does not satisfy the needs. Numerous hydrological studies such as Abdul Aziz and Burn (2006); Burn and Hag Elnur (2002); Burn and Hesch (2007); Burn (2008); Cunderlik and Burn (2002); Douglas et al. (2000); Rivard et al. (2009); Yue et al. (2003) clearly document how important bootstrap in hydrology is, especially in conjunction with the trend analysis, and its potential has not definitely been exhaustively exploited in current times when fast computers are easily accessible. Because there was a method proposed by the author of the thesis based on bootstrapping in paper A4, it is advisable to briefly look at its bases here.

The bootstrap is based on a high number of calculations on permuted data. For illustration purposes, denote an available dataset as $x_k = \{x_1, x_2, \dots, x_K\}$ and assume that it creates a random sample (i.e. observations are independent and identically distributed). There are almost no restrictions on the distribution of the data. Assume further we want to estimate a parameter of the distribution denoted as θ. Denote its counterpart as $T(x) = T(x_1, x_2, \dots, x_K)$. The simplest situation is to estimate the expectation, $\theta = \mathrm{E}(x_k)$, which can be estimated by sample mean $\overline{x}$ as

$$\overline{x} = \frac{1}{K}\sum_{k=1}^{K} x_k \tag{3.84}$$

The idea of the basic bootstrap can be described in four steps

1. From the original dataset x_k generate a random sample $y^* = y_1, y_2, \dots, y_\ell$ where $\ell < K$.

2. Calculate an estimate of the required statistic based on the sample y^* and denote it as $T(y^*)$.

3. Repeat steps (1) and (2) B times where B is a large number (e.g. $B = 1000$ or $B = 10000$).

4. Use the B sample statistics $T(y_1^*), \dots, T(y_B^*)$ to estimate the sample distribution and other statistical properties (like mean, variance, confidence intervals, quantiles, etc.) of the initial estimate $T(x)$.

It is proved that the bootstrap empirical distribution function is a consistent estimator of the distribution function of the initial estimate $T(x)$ (Petrásek, 2008). However, when we want to apply the bootstrap technique to time series data, we meet the problem that the data are not independent, and when taking usual random sample from the initial time series x_k the nature of the dependence disappears.

To solve this problem, among others, the so-called *block bootstrap* was developed during which, when taking the bootstrap random sample, we do not sample particular observations but rather we sample whole parts (i.e. blocks) of the initial series, and so the nature of the dependence is still kept. The algorithm is as follows

1b. Circle the initial time series so that we do not work with a finite sample $x_1, \dots, x_K$ but rather we have and independent series

$$x_1, \dots, x_{K-1}, x_K, x_1, x_2, \dots, x_{K-1}, x_K, x_1, x_2, \dots$$

2b. Define the blocks Z_ℓ from the initial sample given by its initial point $x_{(k_\ell)}$ and length b_ℓ. So then $Z_\ell = \{x_\ell, x_{(\ell+1)}, x_{(\ell+2)}, \dots, x_{(\ell+b_\ell-1)}\}$. If the value $\ell + j$ is greater than K, then replace $\ell + j$ by $\ell + j - K$. The initial points $x_{(k_\ell)}$ are selected randomly from the univariate distribution on $\{x_1, \dots, x_K\}$. The lengths of the blocks are also selected randomly from some distribution $F_b(\cdot)$ depending on a parameter b specified later.

3b. Generate the bootstrap sample from the random blocks $Z_1, \dots, Z_K$ of length j, so that $b_1 + b_2 + ... + b_j$ is the smallest integer greater than K. The bootstrap sample can be then written as

$$y^* = Z_1, Z_2, \dots, Z_j =$$

$$= x_{(k_1)}, x_{(k_1+1)}, x_{(k_1+2)}, \dots, x_{(k_1+b_1)}, x_{(k_2)}, x_{(k_2+1)}, x_{(k_2+2)}, \dots, x_{(k_2+b_2)}, \dots$$

4b. Use the first K values from y^* to calculate the sample statistics $T(y^*)$.

5b. Repeat steps (3) and (4) B times where B is a large number (e.g. $B = 1000$).

6b. Use the B sample statistics $T(y_1^*), \dots, T(y_B^*)$ to estimate the sample distribution and other statistical properties of the initial estimate $T(x)$.

Again, it is proved that such sample distributions have very good properties (see Petrásek, 2008). What only remains is the selection of the optimal value of the lengths of the blocks, particularly the parameter b. There are many ways how to choose it. But all of them agree on the rule that the length has to be greater than the order of the serial dependence (significant terms of the autocorrelation function). An automatic selection of b is possible. A combination of this method and the MK trend test resulted in the suggestion of the ABBS–MK test (see Chapter 3.8.2 for further details)

3.7.8 A note on the need for parametric trend tests

Of course, nonparametric techniques based on the MK test can only suggest whether the trend is significant or not. Nevertheless, when it comes to the modelling of the future, the parametric counterparts become helpful.

The adjusted likelihood ratio test (ALRT) undoubtedly belongs to this valuable group. It was proposed by Cohn and Lins (2005) and is capable of detecting significant linear trends while dealing with possible LTP in a time series. Briefly, its purpose is to distinguish between the models incorporating purely stochastic component that may be correlated and the models incorporating, besides stochastic part, also the deterministic part in terms of the slope of a linear trend. Its test statistic, so-called likelihood ratio, is compared to the χ^2 distribution quantiles to decide if the model without the trend component (i.e. the null hypothesis) is better than the model with it (i.e. the alternative hypothesis). Although the theory of likelihood ratio test is known, for instance, in climatology Wilks (2011), the novelty of the test suggested by Cohn and Lins (2005) lies in the fact that it adjusts the bias in the estimation of the parameters such as the fractional differencing parameter d that is quantified simultaneously with the slope of the deterministic trend. The adjustment, however, is based on Monte Carlo simulations that were performed solely for the range $d \in (0, 0.5)$, which may cause problems when data reveal antipersistence, as found by the author of the thesis during his experiments. Nevertheless, despite this downside, the test deserves it attention without a doubt.

3.8 Modifications of trend tests proposed by the author

During his PhD study, the author also proposed new statistical test devoted to the detection of trends in hydrometeorological time series that may feature persistence. A brief description of three such techniques follows, of which the first two were published in papers A5 and A4. The last one was suggested only during a conference where a poster presentation was exhibited.

3.8.1 Regional TFPW–MK test

In paper A5, a new method dedicated to the detection of field significant long-term monotonic trends was proposed for the case of the series composed of well-spring discharges (otherwise called spring yields). Building on the findings of (Douglas et al., 2000) and Yue and Wang (2002b), the TFPW–MK test capable of dealing with STP in the series measured at sites was taken and extended to regions. This was done through a so-called regional MK test statistic that in fact represented the sum of all the test statistics resulting from each series pertaining to a region. That is

$$SR = \sum_{p=1}^{P} S_{MKp} \tag{3.85}$$

with p denoting the particular well-spring. When dealing with regional trends, the so-called cross-correlation poses another problem. If the series within a region were not cross-correlated, the variance of the regional test statistic would be a simple sum of variances of every single at-site test statistic as follows

$$\sigma_{SR}^2 = \sum_{p=1}^{P} \sigma_{MKp}^2 \tag{3.86}$$

However, the situation is complicated when the cross-correlation is present among data which has effects similar to the serial correlation. It means that the information content in data is actually less than it would be if the positive cross-correlation did not take its effect. Therefore, similar to the autocorrelation, here the variance must be corrected too. In paper A5, it was carried out according to

$$\tilde{\sigma}_{SR}^2 = \sum_{p=1}^{P} \sigma_{MKp}^2 + 2\sum_{p=1}^{P-1}\sum_{\tilde{p}=1}^{P-p} \mathrm{cov}(S_{MKp}, S_{MKp+\tilde{p}}) \tag{3.87}$$

with the cov(·) function defined similarly as in Eq. 3.7, but this time for the MK statistics S_{MK}. Due to the nontrivial quantification of these covariances caused by the possible presence of ties (e.g. Dietz and Killeen, 1981), the right hand side Eq. 3.87 had to be rewritten so that the PMCCs among pre-whitened (due to missing values overlapping) series were necessary (Yue and Wang, 2002b)

$$\tilde{\sigma}_{SR}^2 = \sum_{p=1}^{P} \sigma_{MKp}^2 + 2\sum_{p=1}^{P-1}\sum_{\tilde{p}=1}^{P-p} \sqrt{\sigma_{MKp}^2\sigma_{MKp+\tilde{p}}^2}\,\mathrm{corr}(x_{k,p}, x_{k,p+\tilde{p}}) \tag{3.88}$$

where, again, indexes p and $\tilde{p}$ indicated the well-springs such that $p \neq \tilde{p}$. Note that the series x_k must be pre-whitened (i.e. they are by-products of the TFPW–MK test) before the computation of correlations corr(·).

The regional variance from Eq. 3.88 may then be used for the standardization of the regional TFPW–MK test statistic SR, which results in the statistic Z_{SR} that obeys the standard Gaussian distribution whose quantiles can serve as critical values when deciding if the Z_{SR} is significantly different from zero. That is

$$Z_{SR} = \begin{cases} \dfrac{S_{SR}-1}{\sqrt{\tilde{\sigma}_{SR}^2}} & \text{if } S_{SR} > 0 \\ 0 & \text{if } S_{SR} = 0 \\ \dfrac{S_{SR}+1}{\sqrt{\tilde{\sigma}_{SR}^2}} & \text{if } S_{SR} < 0 \end{cases} \tag{3.89}$$

The method needs at least two objects (i.e. $P = 2$) measuring within the regions of interest. It is quite similar to the procedure published in Yue and Wang (2002b) or Yue et al. (2012). Two aspects, however, makes the regional TFPW–MK test different from that of Yue and Wang (2002b). First, it is based on the TFPW–MK test that is easily accessible through the `zyp` `R` package, second, in takes into account the ties that definitely play a crucial role in groundwater hydrology.

3.8.2 Automatic block bootstrap MK test

In paper A4 the author uses a test based on the block bootstrap with automatically selected block lengths and calls it the ABBS–MK test. The target of the method was to select the critical value for the MK test statistic S_{MK} defined in Eq. 3.43. The procedure was as follows

1. Define random blocks from circulated time series according to steps (1b) and (2b) as described in Chapter 3.7.7 with automatically selected block lengths.
2. Generate a bootstrap sample y^* according to step (3b) in Chapter 3.7.7.
3. Calculate the MK statistics $S_{MK}(y^*)$
4. Repeat the steps (2) and (3) a large number of times (e.g. $B = 1000$).
5. Use the B bootstrap estimates to define the 95% confidence interval for the MK test statistic with the lower limit = 2.5% sample quantile and the upper limit = 97.5% sample quanile. The quantiles can be obtained by the percentile method described, for instance, in Davison and Hinkley (1997).
6. If the original MK test statistic $S_{MK}(x)$ falls outside the confidence limits, then reject the null hypothesis of no trend.

A test defined this way was already used in Khaliq et al. (2009b) or in Önöz and Bayazit (2012). In the first of these two papers it is derived that the optimal block length b should be $b = h' + C$, where h' is the number of significant autocorrelation coeficients and C is a constant. In most cases, it is enough to take $C = 1$ but the optimal value of C with respect to the type I error can be obtained only via simulation as done in Önöz and Bayazit (2012).

Due to a high number of applications of the test in paper A4 (particularly discharge series from 144 water-gauging stations were examined plus several different time periods and indicators were inspected), it was desired to provide the selection of block lenth b fully automatically. Due to this requirement the author decided to use the procedure defined in Politis and White (2004) and Patton et al. (2009) which is implemented in the `np` `R` package as the function `b.star()` (Hayfield and Racine, 2008). The selection of the block length is here based on the minimization of the mean squared error of the estimate of the following variability parameter

$$\sigma^2_{BBS} = \lim_{K\to\infty} \mathrm{var}(\sqrt{K}\overline{x}) \tag{3.90}$$

which corresponds to the target of the paper: estimate the distribution of sample mean. Hence, it is clear that such selected block length is not optimal for the MK statistic, but since it satisfied the requirement that $b = h' + C$, where C is a nonzero constant, it is sufficient for the test and optimal for the practical application of the method in paper A4.

For completeness, it should also be specified that when selecting the block-length, the stationary block bootstrap version was chosen when conducting the computations for paper A4. In that case, the optimal block length b is defined as

$$b = \left(\frac{2\Gamma^2}{\Delta}\right)^{1/3} K^{1/3} \tag{3.91}$$

where

$$\Gamma = \sum_{h=-\infty}^{\infty} |h|\gamma(h) \tag{3.92}$$

and

$$\Delta = 2 \sum_{h=-\infty}^{\infty} \gamma(h) \tag{3.93}$$

Furthermore, $\gamma(h)$ denotes the autocovariance function at lag h.

3.8.3 Generalized equivalent sample size MK test

As was outlined earlier, there are modifications of the MK test dealing with STP and LTP separately by means of the equivalent sample size approach. It would be therefore interesting to look at the performance of a test in fact representing a combination of these two modifications. In late 2015, the author of the thesis applied a prototype of this test to the drought-related time series other than in papers A3 and A4.

Basically, first, it is necessary to decide if a long memory or short memory process should model the stochastic component of a series. It can be conducted via the Hurst exponent estimation and, subsequently, its testing for significance. If this exponent turns out to be different from the value of 0.5, the modification described in Chapter 3.7.4 should be applied. Otherwise, the test should switch to a modification accounting for STP such as that described in 3.7.2.

However, as was said, this type of test has been suggested very recently and its examination is still at its early stages. Mainly, the significance of the Hurst exponent should be considered together with the uncertainty in its estimation.

3.9 Other aspects addressed in papers

Many other aspects were addressed in attached papers. Mainly, the author devoted his attention to extremes in precipitation series in terms of frequency analysis in papers A7–A9. However, in spite of their importance, the indicators of maximum daily precipitation were somewhat secondary and they rather served as a basis for trend analysis that proved to be a good tool when investigating if nonstationary models should be fit to these data (see paper A8).

4. Results – peer-reviewed papers

The thesis is based on nine attached peer-reviewed papers or papers under review. For the findings to which the research work of the candidate contributed, kindly skip to the attachments, that is Chapter 6.

5. General discussion

Two clearly discernible aspects meet in this thesis that should be discussed generally. This may help readers overcome the fact that things are somewhat scattered throughout the attached papers and help them follow some synthesis resulting from the years spent by the author with his study. Among the aspects definitely belong the methodology used in the papers and the findings regarding the changes in the Czech environment, especially its water resources and climate. Both may be beneficial either to mathematicians dealing with applications of their methods to environmental data or to geoscientists who need such methods for their practical purposes.

5.1 Discussion of methods

As can be seen from the huge list of the methods described in preceding chapters, though a lot of them having been applied by the author on his own, it is really not easy to synthesize results revealed by them. In fact, not identical methodology was applied throughout all the papers, which is somewhat typical for research conducted during the involvement in several different scientific projects dealing with various issues. Also, the development of the author as a scientist in terms of his discoveries of better methods from literature in later times of his study played an important role. Therefore, here, the discussion should be considered rather as a list of planned activities of the author alone or as a recommendation to others who would like to continue with this research.

For instance, the methods used for the data imputation may serve as a good example of what has just been mentioned. Whereas, in papers A1 and A2 missing values were filled in using mainly the MLR technique performed without a batch in the times of author's early scientific work, in papers A8 and A9, the data were estimated via the technique programmed by Xu (2016) that was implemented in statistical software. Moreover, the reason why different techniques were utilized in papers A8 and A9 on one hand and in paper A7 on the other, was rather a delay paradoxically caused by the Czech climatologists who did not want to provide the Polish colleagues with 'such a huge amount of data' without another accompanying bilateral contract, which would have led to further delay that the authors could not afford. It is true that the Polish colleagues had their own software prepared incorporating the TIN approach (see Chapter 3.1.4), but their work was finally hindered by the data policy that is applied by the CHMI. From this, it is also evident that it is still not easy to get access to hydrometeorological data from different countries even though being affiliated with a hydrometeorological service, as pointed out in paper A9. However, the question remains why at all the data had to be imputed in some studies and somewhere not. What determined it mainly was the nature of subsequent statistical techniques themselves that required it. Sometimes we simply wanted to avoid biases (caused by spatially interpolated precipitation in A1 and A2) and sometimes the knowledge of the author was not sufficient and rather he accepted the advice from experienced Polish engineers (papers A7–A9). In the future, it would be really interesting to employ some more rigorous technique that preserves both serial and spatial correlation. And these techniques do exist as may be documented by the SSA-based approaches by Kondrashov and Ghil (2006) and v. Buttlar et al. (2014). Notice also that somewhere the imputation techniques were not applied because the MK test for trend is reported to be not much sensitive to the presence of missing values (see e.g. Hirsch et al., 1982; Hirsch and Slack, 1984). Nevertheless, in these cases the selection of data was based on a simple criterion – do not subject to the analyses the series that contain many missing values.

Of course, for trend analyses, it will always be more comfortable to use data that were pre-processed by somebody else, as was the case of technical precipitation amounts investigated in paper A6. On the contrary, it is not advisable to use these data without metadata or the knowledge of how they came into being or for what purposes. For instance, in spite of being devised in paper A6, it finally turned out that these data are not appropriate for assessing extremes. Rather, only mean levels of precipitation should be evaluated through these data since the extremes in fact disappeared from technical series by averaging, as pointed out by Valeriánová (2016, personal communication). This was the rationale for our own pre-processing of the precipitation data in papers A7–A9, thanks to which the extremes were preserved.

The same actually applies for software written by somebody else. It was found in paper A9 empirically that the function `zyp.yuepilon` in the `R` package `zyp` (i.e. TFPW–MK test; Bronaugh and Werner, 2013), although being very popular, seemingly does not test the significance of the lag-one autocorrelation coefficient. During the careful inspection of the code, which is possible as regards many `R` packages after some investigation of how to do it under the MS Windows operating systems, it was also found that the function does not check the significance of the Sen slope estimator from Eq. 3.44 and that the authors want to improve it in the future. Therefore, it is highly advisable to use software were one can look into the code and can modify it according to his/her needs, which `R` definitely fulfills. Note, however, that in their original paper about the TFPW–MK test, Yue et al. (2002b) did not require the testing of the Sen slope significance.

From studies focused on a few river basins from Czechia (A1 and A2), it is clear that the process of comparing the climate variables with those representing the time series of discharge has not been properly finished yet, at least as regards the use of more suitable method exactly devised for these purposes nowadays. Namely, in these papers, somewhat obsolete methods or the methods used today for the initiation of more thorough investigation were utilized such as the tests for the presence of abrupt changes that, based only on statistical properties of time series without having the metadata, gave rather mixing results regarding times of changes. Thus, especially, the wavelet analysis along with its cross-wavelet spectra has a great potential in order to compare precipitation with discharge as done in Szolgayova et al. (2014b) who made use of the methods outlined in Torrence and Compo (1998) and later works. The findings resulting from such application would definitely help one decide also on the lags between the rainfall events and the runoff response in different basins. Furthermore, building on this, a suitable stochastic model would be possible for these basins that would incorporate also external variables such as precipitation.

Another issue is connected with data accessibility caused not only by data policies. As the author worked primarily on the Rolava River basin in the Ore

Mountains (see papers A1 and A2), it was found that exactly this area suffers from a sparse hydrometeorological network of stations whose data contain a lot of missing values and the techniques devoted to imputation are highly advisable here. Note that this is not only the case of the Czech part of the Ore Mountains but also their German part. For CC studies, therefore, it would be beneficial to prepare some sound time series, specifically composed of precipitation. A combination of ground observations and radar images as done, for in stance, in Haberlandt (2007); Šalek (2000), but not necessarily for hourly data, offers its potential, but one must bear in mind the distance of the Ore Mountains from the closest radar located in the Brdy hills south of Prague is relatively high and poses a risk of biases.

At the beginning of 2016, the author of this thesis prepared his own dataset comprising precipitation or areal precipitation totals for several river basins in the Ore Mountains that are characterized by longer observations of discharge (i.e. series starting at least in the 1960s). The dataset was prepared by the use of the author's own universal kriging technique written in `R` that incorporates the relationships between precipitation and other covariates (elevation, geographical coordinates) in terms of a MLR in which the important covariates are searched via the Akaike Information Criterion (AIC; Konishi and Kitagawa, 2008). This dataset, building on the ground observations from Czechia as well as Germany, may reveal some important features in the future, especially in conjunction with the wavelet analysis mentioned above.

On the other hand, in nation-wide studies performed in papers A3, A4, A5 and A6, certainly, there has been an open space for what is called *false discovery rate* (FDR; Benjamini and Hochberg, 1995). Especially, the numbers of some of the significant trends regarding selected hydrometeorological variables in papers A4, A5 and A6 brings us to the question if they are not the result of a chance because their rate is very close to the proportion of erroneously detected trends allowed by the type I error. In papers A4 and A6, moreover, two significance levels were studied, which may be a good starting point for another analysis. For example, despite not the same techniques applied, the numbers of trends identified in paper A4 are not much different from those published in Fiala et al. (2010), the first author of which in his thesis (Fiala, 2011) concludes essentially with the same thought. The theory of FDR is known in climatology (Ventura et al., 2004; Wilks, 2006) and hydrology as well (e.g. Khaliq et al., 2009b), but, to the author's knowledge, it has not been utilized in Czech hydrology, at least concerning the trend analysis.

After the publication of the papers of Milly et al. (2008), a rise of the interest in the nonstationary probabilistic models was perceptible especially outside the territory of Czechia. In paper A8, it is shown that the trend analysis may be beneficial as regards the decision on whether to use stationary or nonstationary models such as point processes that have been developed since the 1980s (see e.g. Gilleland and Katz, 2014). However, in 2015, a paper from Serinaldi and Kilsby (2015) appeared that, on the other hand, warns against the use of nonstationary models because of the sampling uncertainty and the uncertainty in the time-dependent parameters estimation. Rather, much simpler stationary models should be preferred as done in papers A7 and A9, despite some trends seemed to be significant.

In papers A4 and A5, some new trend tests were proposed by the author of the thesis. While in paper A5, a new technique for detecting the regional trends, making use of the TFPW–MK test of Yue et al. (2002b), was suggested (see Chapter 3.8.1), in paper A4, the so-called ABBS–MK test based on the block bootstrap was developed using the `R` package `np` (Hayfield and Racine, 2008) whose function `b.star()` determines an optimal block length. Wisely inserted into the `ts.boot()` function from the `boot R` package (Canty and Ripley, 2016) together with the `MannKendall()` function from the package `Kendall` (McLeod, 2011), it can be used for obtaining the quantiles via the percentile method (e.g. Davison and Hinkley, 1997). Although the optimizing process is based on the statistic that is not in accordance with the MK test statistic, the proposed ABBS–MK test suffices for practical purposes and can be utilized especially when a batch processing is needed (as was the case for 144 water-gauging stations in paper A4). For detailed explanation, see Chapter 3.8.2.

Yet, another trend test was suggested by the author, although having been not published so far. The so-called GESS–MK test could be utilized when one is uncertain if STP or LTP contaminate the time series under investigation. If there would be a proper method for testing the significance of the Hurst exponent, considering simultaneously the uncertainty in its estimation, the equivalent sample size modifications of the MK test (see Chapters 3.7.2 and 3.7.4) might switch from one to another just according to the Hurst exponent significance. However, this type of test is still in development because the author's `R` code now requires too much computational capacity.

Last but not least, something should be specified concerning the Wilcoxon signed rank test (see Chapter 3.3.1) applied in paper A9. The data that could be divided into two halves allowed it to be utilized in similar way as advised in Lettenmaier (1976) who worked with the parametric t-test. The rationale of designing the test like that was to overcome the possible presence of persistence in time series of precipitation maxima. Nevertheless, it is clear that this was only an attempt to find out if there are some significant differences between the former (older) and the latter (newer) parts of the series. In fact, no change point was known in advance where rather the Wilcoxon rank sum test 3.2.1 might be applied in most of the cases, despite the risk of working with correlated data.

5.2 Discussion of results

Without proper (e.g. Monte Carlo) simulation experiments, it is difficult to conclude which of the trend test is best. It is true that some of these attempts were carried out in hydrology as regards the modifications of the MK test, but they did not include all of the tests used here (see e.g. Khaliq et al., 2009b; Khaliq and Sushama, 2012). The simulations together with the improvements (in terms of the insensivity to STP or LTP) are one of the aspects that should be addressed in the future, similar to the examination of the CS test that actually cannot be recommended in hydrology (Ledvinka, 2014b). Notwithstanding, some remarks on the findings appearing in the attached papers themselves can be made.

In essence, papers A1 and A2 with selected mountainous river basins confirm what has been summarized by Fiala (2011) who wrote about the European context. The winter discharges are increasing due to rising air temperature and precipitation which, on the contrary, decreases in the Vydra River basin in the Bohemian Forest in late spring (but in April elsewhere as well, see paper A6). This means the spring declines in precipitation manifest themselves only somewhere in decreasing discharge in these months. Rather, the shifts towards earlier snowmelt along with changes of the number of days with snow cover stand behind the decreases in discharge in late spring, as shown for the Jizera Mountains by Vajskebr et al. (2013) as well. This effect is documented also by the yields of well-springs located in the Bohemian Forest (see paper A5). Moreover, the 7-day low flows investigated in paper A4 or Fiala et al. (2010) react similarly. Namely their timing shifts towards earlier dates are apparent regardless if they are observed in the mountain or in the lowland areas, which is corroborated by the fact that, unlike in Fiala et al. (2010), no increasing trends were found in the series composed of Julian days (see paper A4). Seemingly, the basins in northern Moravia experience different changes than the Bohemian ones.

Furthermore, according to papers A1 and A2 as well as Fiala (2011), overall, the air temperature has been rising mainly since the early 1980s. Notably, the temperature rises in winter (December and January) and summer. The winter increase, of course, causes the snow cover to melt earlier, and the precipitation to fall more and more often in the liquid form which in turn again causes the snowmelt.

The mentioned facts are closely associated with the patterns of precipitation. In paper A6 it is emphasized that, if some trends were found, they help one distinguish that Bohemia is getting wetter, while Moravia becomes drier, which corresponds with the long-term course of drought-related indicators (paper A4) or trends in well-spring yields (paper A5), where declines, if some, are apparent notably in the southeast and east of Moravia. The exception may be the hydrogeological regions situated southwest of the city of Prague (e.g. along the Berounka River). Furthermore, one can observe that mountains or mid-altitudes reveal some increases in precipitation, which may pose some problems in future water management due to more frequent flooding especially at the mid-altitudes (Elleder, 2016, personal communication). Also, some changes are apparent in the annual hyetographs where shifts of maximum monthly precipitation from June to July were found. The shapes of the hyetographs change as well, indicating more uneven distribution of precipitation throughout the year in the present days, which may be the reason why in some river basis in Czechia the changes in seasonality can be found as well (e.g. Jarušková et al., 2015).

Notwithstanding, it is more than evident that the numbers of significant trends in all the variables investigated are somewhat rare in comparison to the numbers of stations that were assessed in nation-wide studies. Although different time periods were chosen for the investigation in different papers, maybe the fact that the time series started predominantly after 1960 can play and important role. As pointed out by Vlnas and Fiala (2010), the inclusion of drastic droughts that occurred in 1947 and 1953/1954 would definitely flip the directions of trends (e.g. increasing deficit volumes in the period 1961–2005 would change to the decreasing ones). It is hard to conclude without longer series available.

Also, having longer series would precise the findings regarding the numbers of long memory processes probably underlying the time series of 7-day low flows or well-spring yields. Besides this, the presence of some unit root processes is very interesting, indicating rather the erroneous measurements than reality. For instance, their concentration, regarding the well-spring yields, along the Lower Jizera River reaches looks suspicious. Moreover, Vlnas (2015) stressed that decreasing patterns in the yields in northern Bohemia are rather of measurement origins as well. Nevertheless, Ledvinka (2014a) had a daily discharge series from the Ore Mountains that starts in the 1920s and, by means of this and other discharge series, found that in the small catchments located there the, Hurst phenomenon is evident, regardless of the theoretical suggestion made by Mudelsee (2007) who states that the probability of the Hurst phenomenon occurrence in river discharge increases with the area of the basin or with the river network complexity. Definitely, this issue would deserve its attention as well as the issue of multifractality in hydrology that needs not be evident from monthly series, but daily time scale allows it to be found (see e.g. Fig. 3.2 or for the case of air temperature Fatichi et al., 2009). Also, Kantelhardt et al. (2006) emphasize some similar patterns concerning precipitation. Exactly this may be the reason why Szolgayova et al. (2014b) found different patterns regarding daily and monthly precipitation.

In papers A1 and A2, it is stated that the snow disappears mainly in lower parts of the basins. This statement, however, should be confirmed similarly as in the case of the deficit volumes. Namely, it is necessary to standardize these and other indicators in order to allow the comparison among different basins with various flow rates as well as various amounts of snow. The need for standardization regarding deficit volumes was discussed, for instance, in Vlnas and Fiala (2010). After the standardization, more realistic relationships between them and physiographic features of the basins were uncovered, which has not been performed by the author of the thesis properly yet, despite some attempts do appear in papers A5 and A6 as regards the trends in drought-related indicators and precipitation. Also, more sound analysis is planned concerning the Hurst exponent and several physiographic features of Czech river basins as done, for instance, in Koscielny-Bunde et al. (2006). Notably, according to Daňhelka (2014, personal communication), a more appropriate investigation of the hydrometeorological data representing the Jeseníky Mountains would be interesting since the technical reference precipitation series apparently reveal patterns significantly different from measured discharges. However, first it is necessary to gather a proper database composed of various data relating to the Czech environment, which might be a problem. Namely, in spite of the fact that Czech climatologists have some dataset suitable for CC studies, as regards at least mean levels, Czech hydrologists do not have such dataset, which is caused, among others, by the anthropogenic impacts occurring in many river basins in Czechia.

Papers A7–A9 seem to be standing aside, but it is not true. Clearly, using the trend analysis, it is shown there that also the rainfall maxima reveal hardly any changes, which is not only the case of the territory of Czechia. Therefore, they simply confirm the findings that the precipitation amounts are rather stable in the Central European region which is, from a wider point of view, bounded by the regions with increases in the north and decreases in the south (e.g. Stahl et al., 2010).

6. Conclusions

In this thesis, which is composed of nine attached papers or manuscripts, mainly the trend analysis of various hydrometeorological variables representing the territory of Czechia and its surroundings was performed. For this purpose, a lot of tests for gradual trends and many other statistical tools were used. Primarily, `R` statistical software whose specific language is easy to adopt, which in turn allows to write various scripts very quickly (and sometimes instantly after reading papers), proved to be a good working environment for the author as a statistical hydrologist. In fact, the trend analysis here was originally intended as a preliminary activity before building a proper stochastic model with external variables, such as groundwater and precipitation (or climate in general), and capable of predicting future development of daily discharge series for some selected river basin from Czechia. However, the issue of trend analysis is now so popular in hydrology, especially in conjunction with the question of climate change and its impacts on water resources, that it was almost impossible to leave this topic not being influenced by the flood of literature dealing with it and look also at other fields of hydrology that are more close to real modelling.

During the past, specifically in hydrology, several modifications of trend tests were developed in order to overcome the issue of persistence contaminating hydrometeorological data series. Two types of persistence have to be distinguished – short-term persistence and long-term persistence. Briefly, the presence of persistence in time series means that expressions for the variances of classical test statistics do not hold true because the assumption of independence among data is violated. Exactly this was the main reason for developing the modifications described in this thesis, from which some new techniques were proposed by the author alone.

It can be concluded that exclusively short-term persistence tests were utilized in practical applications here. The analyzed data mainly represented the time period starting from 1960s which, from a certain point of view, cannot be considered long, as the title of the thesis suggests. However, even this length of data allowed the author to gain some knowledge of the dynamics of the relationships between climate and water resources virtually in the whole of Czechia since several nation-wide studies were conducted among others. Besides this, via a unit root test and the stationarity test accompanied by the Hurst exponent estimation, an examination of several series for the presence of long-term persistence was carried out. The findings imply that long-term persistence may be responsible for some falsely detected trends in hydrological time series such as drought-related indicators (mainly in SW and NE Czechia) or well-spring yields. On the contrary, the precipitation series may be characterized by short-term persistence or white noises (sometimes about deterministic trends), which is in accordance with literature from abroad (e.g. Kantelhardt et al., 2006).

According to the facts revealed by this thesis, and excluding the rising air temperature or disappearing snow, it seems that a stationary predictive model would be sufficient for the variables studied because not many places in Czechia showed significant deterministic trends. However, a lot of investigation remains to be done before establishing this statistical model. If possible, it would be better to conduct this in scientific teams because there is no chance for an individual to master all the statistical methods encoutered in hydrology.

Last but not least, it should be noted that in Czechia and Slovakia, the modified trend test accounting for persistence became known with considerable delay, essentially at the turn of the 2000s and the 2010s as stated in paper A4. However, despite the distribution-free Mann–Kendall test is already quite famous here, there are still some hydrologists who do not take into account the issue of persistence at all (see e.g. Blahušiaková and Matoušková, 2015; Zeleňáková et al., 2012, 2014a,b, 2015a,b). Moreover, in Czechia, there are still some geophysicists who, in an attempt to analyze hydrometeorological time series, use the OLS linear regression even though the assumptions are not met. They even do not test the significance of the slope parameter of the regression and rather report only on tendencies given by the lines in graphs (see e.g. Střeštík et al., 2014). Therefore, the author strongly believes that the visibility of his thesis will substantially contribute to the reduction of such studies. Also, the release of a new `R` package would definitely be helpful in this process, which is really possible in the near future because its preparation was in fact already initiated by this research.

Bibliography

Abdul Aziz, O. I. and Burn, D. H. (2006). Trends and variability in the hydrological regime of the Mackenzie River Basin. *Journal of Hydrology*, 319(1-4):282–294.

Adamowski, K., Prokoph, A., and Adamowski, J. (2009). Development of a new method of wavelet aided trend detection and estimation. *Hydrological Processes*, 23(18):2686–2696.

Alexandersson, H. (1986). A homogeneity test applied to precipitation data. *Journal of Climatology*, 6(6):661–675.

Alexandersson, H. and Moberg, A. (1997). Homogenization of Swedish temperature data. Part I: Homogeneity test for linear trends. *International Journal of Climatology*, 17(1):25–34.

Anderson, R. L. (1942). Distribution of the serial correlation coefficient. *The Annals of Mathematical Statistics*, 13(1):1–13.

Banerjee, A., Dolado, J. J., Galbraith, J. W., and Hendry, D. F. (1993). *Co-Integration, Error Correction, and the Econometric Analysis of Non-Stationary Data.* Advanced Texts in Econometrics. Oxford University Press, Oxford.

Barbosa, S. M. (2011). Testing for deterministic trends in global sea surface temperature. *Journal of Climate*, 24(10):2516–2522.

Barbosa, S. M., Scotto, M. G., and Alonso, A. M. (2011). Summarising changes in air temperature over Central Europe by quantile regression and clustering. *Natural Hazards and Earth System Sciences*, 11(12):3227–3233.

Barbosa, S. M., Silva, M. E., and Fernandes, M. J. (2008). Time series analysis of sea-level records: characterising long-term variability. In Donner, R. V. and Barbosa, S. M., editors, *Nonlinear Time Series Analysis in the Geosciences: Applications in Climatology, Geodynamics and Solar-Terrestrial Physics*, number 112 in Lecture Notes in Earth Sciences, pages 157–173. Springer-Verlag, Berlin.

Bayley, G. V. and Hammersley, J. M. (1946). The "effective" number of independent observations in an autocorrelated time series. *Supplement to the Journal of the Royal Statistical Society*, 8(2):184–197.

Bažatová, T. and Šimková, J. (2015). Changes in runoff regime. The Lomnice catchment case study. *Soil and Water Research*, 10(1):40–48.

Benjamini, Y. and Hochberg, Y. (1995). Controlling the false discovery rate: a practical and powerful approach to multiple testing. *Journal of the Royal Statistical Society. Series B (Methodological)*, 57(1):289–300.

Beran, J. (1994). *Statistics for Long-Memory Processes.* Number 61 in Monographs on Statistics and Applied Probability. Chapman & Hall, New York.

Beven, K. (2012). *Rainfall-Runoff Modelling: The Primer.* Wiley-Blackwell, Chichester, West Sussex ; Hoboken, NJ, 2nd edition.

Blahušiaková, A. and Matoušková, M. (2015). Rainfall and runoff regime trends in mountain catchments (Case study area: the upper Hron River basin, Slovakia). *Journal of Hydrology and Hydromechanics*, 63(3).

Blöschl, G., Sivapalan, M., Wagener, T., Viglione, A., and Savenije, H., editors (2013). *Runoff Prediction in Ungauged Basins: Synthesis across Processes, Places and Scales.* Cambridge University Press, New York.

Box, G. E. P., Jenkins, G. M., and Reinsel, G. C. (2008). *Time Series Analysis: Forecasting and Control.* Wiley Series in Probability and Statistics. John Wiley & Sons, Hoboken, NJ, 4th edition.

Brockwell, P. J. and Davis, R. A. (1991). *Time Series: Theory and Methods.* Springer Series in Statistics. Springer, New York, NY, 2nd edition.

Bronaugh, D. and Werner, A. (2013). zyp: Zhang + Yue-Pilon trends package.

Brunetti, M., Maugeri, M., and Nanni, T. (2001). Changes in total precipitation, rainy days and extreme events in northeastern Italy. *International Journal of Climatology*, 21(7):861–871.

Buishand, T. A. (1982). Some methods for testing the homogeneity of rainfall records. *Journal of Hydrology*, 58(1-2):11–27.

Burn, D. H. (2008). Climatic influences on streamflow timing in the headwaters of the Mackenzie River Basin. *Journal of Hydrology*, 352(1-2):225–238.

Burn, D. H. and Hag Elnur, M. A. (2002). Detection of hydrologic trends and variability. *Journal of Hydrology*, 255(1-4):107–122.

Burn, D. H. and Hesch, N. M. (2007). Trends in evaporation for the Canadian Prairies. *Journal of Hydrology*, 336(1-2):61–73.

Canty, A. and Ripley, B. (2016). boot: Bootstrap R (S-Plus) Functions.

Chen, H.-L. and Rao, A. R. (2002). Testing hydrologic time series for stationarity. *Journal of Hydrologic Engineering*, 7(2):129–136.

Cipra, T. (2008). *Financial Econometrics.* Ekopress, Praha.

Cohn, T. A. and Lins, H. F. (2005). Nature's style: naturally trendy. *Geophysical Research Letters*, 32(23):L23402.

Conrad, V. (1946). *Methods in Climatology.* Harvard University Press, Cambridge, Massachusetts.

Constantine, W. and Percival, D. (2013). wmtsa: Wavelet Methods for Time Series Analysis.

COSMT (2014). *Hydrological Data on Surface Water.* Number ČSN 75 1400 in Czech Technical Standard. Czech Office for Standards, Metrology and Testing, Praha.

Couillard, M. and Davison, M. (2005). A comment on measuring the Hurst exponent of financial time series. *Physica A: Statistical Mechanics and its Applications*, 348:404–418.

Cowpertwait, P. S. P. and Metcalfe, A. V. (2009). *Introductory Time Series with R*. Use R! Springer, Dordrecht; New York.

Cox, D. R. and Stuart, A. (1955). Some quick sign tests for trend in location and dispersion. *Biometrika*, 42(1/2):80–95.

Cunderlik, J. M. and Burn, D. H. (2002). Local and regional trends in monthly maximum flows in southern British Columbia. *Canadian Water Resources Journal*, 27(2):191–212.

Daubechies, I. (1992). *Ten Lectures on Wavelets*. Number 61 in CBMS-NSF Regional Conference Series in Applied Mathematics. Society for Industrial and Applied Mathematics, Philadelphia, Pa.

Davison, A. C. and Hinkley, D. V. (1997). *Bootstrap Methods and Their Applications*. Cambridge University Press, New York.

Dickey, D. A. and Fuller, W. A. (1979). Distribution of the estimators for autoregressive time series with a unit root. *Journal of the American Statistical Association*, 74(366a):427–431.

Dietz, E. J. and Killeen, T. J. (1981). A nonparametric multivariate test for monotone trend with pharmaceutical applications. *Journal of the American Statistical Association*, 76(373):169–174.

Douglas, E. M., Vogel, R. M., and Kroll, C. N. (2000). Trends in floods and low flows in the United States: Impact of spatial correlation. *Journal of Hydrology*, 240(1-2):90–105.

Efron, B. and Tibshirani, R. (1993). *An Introduction to the Bootstrap*. Number 57 in Monographs on statistics and applied probability. Chapman & Hall, New York.

Ehsanzadeh, E. and Adamowski, K. (2010). Trends in timing of low stream flows in Canada: impact of autocorrelation and long-term persistence. *Hydrological Processes*, 24(8):970–980.

Elleder, L. (2015). Historical changes in frequency of extreme floods in Prague. *Hydrology and Earth System Sciences*, 19(10):4307–4315.

Eslamian, S., editor (2014). *Handbook of Engineering Hydrology: Modeling, Climate Change, and Variability*, volume 2. CRC Press, Boca Raton, Fla.

Fanta, B., Zaake, B. T., and Kachroo, R. K. (2001). A study of variability of annual river flow of the southern African region. *Hydrological Sciences Journal*, 46(4):513–524.

Fatichi, S., Barbosa, S. M., Caporali, E., and Silva, M. E. (2009). Deterministic versus stochastic trends: detection and challenges. *Journal of Geophysical Research*, 114(D18):D18121.

Feder, J. (1988). *Fractals*. Springer US, Boston, MA.

Feng, X., Porporato, A., and Rodriguez-Iturbe, I. (2013). Changes in rainfall seasonality in the tropics. *Nature Climate Change*, 3(9):811–815.

Fiala, T. (2008). Statistical characteristics and trends of mean annual and monthly discharges of Czech rivers in the period 1961-2005. *Journal of Hydrology and Hydromechanics*, 56(2):133–140.

Fiala, T. (2011). *Evolution of deficit volumes and mean and low flows in the Czech Republic*. Dissertation thesis, Faculty of Science, Charles Univeristy in Prague, Praha.

Fiala, T., Ouarda, T. B. M. J., and Hladný, J. (2010). Evolution of low flows in the Czech Republic. *Journal of Hydrology*, 393(3-4):206–218.

Fraley, C., Leisch, F., Maechler, M., Reisen, V., and Lemonte, A. (2012). fracdiff: Fractionally differenced ARIMA aka ARFIMA(p,d,q) models.

Frick, C., Steiner, H., Mazurkiewicz, A., Riediger, U., Rauthe, M., Reich, T., and Gratzki, A. (2014). Central European high-resolution gridded daily data sets (HYRAS): mean temperature and relative humidity. *Meteorologische Zeitschrift*, 23(1):15–32.

Fuller, W. A. (1996). *Introduction to Statistical Time Series*. Wiley Series in Probability and Statistics. Wiley, New York, 2nd edition.

Geweke, J. and Porter-Hudak, S. (1983). The estimation and application of long memory time series models. *Journal of Time Series Analysis*, 4(4):221–238.

Gilleland, E. and Katz, R. W. (2014). extRemes 2.0: an extreme value analysis package in R. *Journal of Statistical Software*, pages 1–49.

Grimaldi, S. (2004). Linear parametric models applied to daily hydrological series. *Journal of Hydrologic Engineering*, 9(5):383–391.

Grolemund, G. and Wickham, H. (2011). Dates and times made easy with lubridate. *Journal of Statistical Software*, 40(3):1–25.

Haan, C. T. (2002). *Statistical Methods in Hydrology*. Iowa State Press, Ames, Iowa, 2nd edition.

Haberlandt, U. (2007). Geostatistical interpolation of hourly precipitation from rain gauges and radar for a large-scale extreme rainfall event. *Journal of Hydrology*, 332(1-2):144–157.

Hamed, K. H. (2008). Trend detection in hydrologic data: the Mann–Kendall trend test under the scaling hypothesis. *Journal of Hydrology*, 349(3-4):350–363.

Hamed, K. H. (2009). Exact distribution of the Mann–Kendall trend test statistic for persistent data. *Journal of Hydrology*, 365(1-2):86–94.

Hamed, K. H. and Rao, A. R. (1998). A modified Mann-Kendall trend test for autocorrelated data. *Journal of Hydrology*, 204(1-4):182–196.

Hameed, T., Mariño, M. A., DeVries, J. J., and Tracy, J. C. (1997). Method for trend detection in climatological variables. *Journal of Hydrologic Engineering*, 2(4):154–160.

Hannaford, J. and Buys, G. (2012). Trends in seasonal river flow regimes in the UK. *Journal of Hydrology*, 475:158–174.

Haslett, J. and Raftery, A. E. (1989). Space-time modelling with long-memory dependence: assessing Ireland's wind power resource (with discussion). *Applied Statistics*, 38(1):1–50.

Havlíček, V., Hanel, M., Máca, P., Kuráž, M., and Pech, P. (2013). Incorporating basic hydrological concepts into genetic programming for rainfall-runoff forecasting. *Computing*, 95(S1):363–380.

Hayfield, T. and Racine, J. S. (2008). Nonparametric econometrics: the np package. *Journal of Statistical Software*, 27(5):1–32.

Helsel, D. R. and Hirsch, R. M. (2002). *Statistical Methods in Water Resources*. Number 04-A3 in Techniques of Water-Resources Investigations of the United States Geological Survey. US Geological Survey, Reston.

Hipel, K. W. and McLeod, A. I. (1994). *Time Series Modelling of Water Resources and Environmental Systems*. Number 45 in Developments in Water Science. Elsevier, Amsterdam.

Hirsch, R. M., Helsel, D. R., Cohn, T. A., and Gilroy, E. J. (1993). Statistical analysis of hydrologic data. In Maidment, D. R., editor, *Handbook of Hydrology*, pages 17.1–17.55. McGraw-Hill, New York.

Hirsch, R. M. and Slack, J. R. (1984). A nonparametric trend test for seasonal data with serial dependence. *Water Resources Research*, 20(6):727–732.

Hirsch, R. M., Slack, J. R., and Smith, R. A. (1982). Techniques of trend analysis for monthly water quality data. *Water Resources Research*, 18(1):107–121.

Hosking, J. R. M. (1981). Fractional differencing. *Biometrika*, 68(1):165–176.

Hosking, J. R. M. (1984). Modeling persistence in hydrological time series using fractional differencing. *Water Resources Research*, 20(12):1898–1908.

Hurst, H. E. (1951). Long term storage capacity of reservoirs (with discussions). *Transactions of the American Society of Civil Engineers*, 116:770–808.

Hyndman, R. J. and Fan, Y. (1996). Sample quantiles in statistical packages. *The American Statistician*, 50(4):361.

Hyndman, R. J. and Khandakar, Y. (2008). Automatic time series forecasting: the forecast package for R. *Journal of Statistical Software*, 27(3):1–22.

Jarušková, D., Horáková, H., and Satrapa, L. (2015). Detection of nonstationarities of several small Czech rivers by statistical methods. *Civil Engineering Journal*, 24(4):Article no. 5.

Jarvis, D., Stoeckl, N., and Chaiechi, T. (2013). Applying econometric techniques to hydrological problems in a large basin: Quantifying the rainfall–discharge relationship in the Burdekin, Queensland, Australia. *Journal of Hydrology*, 496:107–121.

Kantelhardt, J. W., Koscielny-Bunde, E., Rybski, D., Braun, P., Bunde, A., and Havlin, S. (2006). Long-term persistence and multifractality of precipitation and river runoff records. *Journal of Geophysical Research*, 111(D1):D01106.

Kaźmierczak, B. and Kotowski, A. (2015). The suitability assessment of a generalized exponential distribution for the description of maximum precipitation amounts. *Journal of Hydrology*, 525:345–351.

Kendall, M. G. (1970). *Rank Correlation Methods.* Griffin, London, 4th edition.

Khaliq, M. N. and Ouarda, T. B. M. J. (2007). On the critical values of the standard normal homogeneity test (SNHT). *International Journal of Climatology*, 27(5):681–687.

Khaliq, M. N., Ouarda, T. B. M. J., and Gachon, P. (2009a). Identification of temporal trends in annual and seasonal low flows occurring in Canadian rivers: the effect of short- and long-term persistence. *Journal of Hydrology*, 369(1-2):183–197.

Khaliq, M. N., Ouarda, T. B. M. J., Gachon, P., and Sushama, L. (2008). Temporal evolution of low-flow regimes in Canadian rivers. *Water Resources Research*, 44(8):W08436.

Khaliq, M. N., Ouarda, T. B. M. J., Gachon, P., Sushama, L., and St-Hilaire, A. (2009b). Identification of hydrological trends in the presence of serial and cross correlations: a review of selected methods and their application to annual flow regimes of Canadian rivers. *Journal of Hydrology*, 368(1-4):117–130.

Khaliq, M. N. and Sushama, L. (2012). Analysis of trends in low-flow time series of Canadian rivers. In *Hydrologic Time Series Analysis: Theory and Practice*, pages 201–221. Springer Netherlands, Dordrecht.

Kliment, Z. and Matoušková, M. (2006). Changes of runoff regime according to human impact on the landscape. *Geografie*, 111(3):292–304.

Kliment, Z. and Matoušková, M. (2008). Long-term trends of rainfall and runoff regime in upper Otava River basin. *Soil and Water Research*, 3(3):155–167.

Kliment, Z. and Matoušková, M. (2009). Runoff changes in the Šumava Mountains (Black Forest[sic!]) and the foothill regions: extent of influence by human impact and climate change. *Water Resources Management*, 23(9):1813–1834.

Kondrashov, D. and Ghil, M. (2006). Spatio-temporal filling of missing points in geophysical data sets. *Nonlinear Processes in Geophysics*, 13(2):151–159.

Konishi, S. and Kitagawa, G. (2008). *Information Criteria and Statistical Modeling*. Springer Series in Statistics. Springer, New York.

Koscielny-Bunde, E., Kantelhardt, J., Braun, P., Bunde, A., and Havlin, S. (2006). Long-term persistence and multifractality of river runoff records: detrended fluctuation studies. *Journal of Hydrology*, 322(1-4):120–137.

Kottegoda, N. T. and Rosso, R. (2008). *Applied Statistics for Civil and Environmental Engineers*. Blackwell Publishing, Oxford, UK, 2nd edition.

Koutsoyiannis, D. (2003). Climate change, the Hurst phenomenon, and hydrological statistics. *Hydrological Sciences Journal*, 48(1):3–24.

Koutsoyiannis, D. (2010). HESS Opinions "A random walk on water". *Hydrology and Earth System Sciences*, 14(3):585–601.

Královec, V. (2009). *Changes of Rainfall-Runoff Conditions in the Upper Opava River Basin*. Diploma thesis, Faculty of Science, Charles Univeristy in Prague, Praha.

Kulkarni, A. and von Storch, H. (1995). Monte Carlo experiments on the effect of serial correlation on the Mann-Kendall test of trend. *Meteorologische Zeitschrift*, 4:82–85.

Kundzewicz, Z. W. and Radziejewski, M. (2006). Methodologies for trend detection. In Demuth, S., Gustard, A., Planos, E., Scatena, F., and Servat, E., editors, *Climate Variability and Change-Hydrological Impacts*, IAHS Red Book 308, pages 538–549. IAHS Press, Wallingford, Oxfordshire, UK.

Kundzewicz, Z. W. and Robson, A., editors (2000). *World Climate Programme - Water. Detecting Trend and Other Changes in Hydrological Data*. Number 45 in WCDMP. World Meteorological Organization, Geneva.

Kundzewicz, Z. W. and Robson, A. J. (2004). Change detection in hydrological records—a review of the methodology. *Hydrological Sciences Journal*, 49(1):7–19.

Kwiatkowski, D., Phillips, P. C. B., Schmidt, P., and Shin, Y. (1992). Testing the null hypothesis of stationarity against the alternative of a unit root. *Journal of Econometrics*, 54(1-3):159–178.

Kyselý, J. and Domonkos, P. (2006). Recent increase in persistence of atmospheric circulation over Europe: comparison with long-term variations since 1881. *International Journal of Climatology*, 26(4):461–483.

Ledvinka, O. (2008). *Trends of Rainfall and Runoff Regime in the Rolava River Basin*. Diploma thesis, Faculty of Science, Charles University in Prague, Praha.

Ledvinka, O. (2014a). Are there nonstationarities and the Hurst phenomenon in discharge series within the Ore Mountains region? In Brych, K. and Tesař, M., editors, *Hydrology of a Small Basin 2014*, volume 1, pages 287–295. Czech Hydrometeorological Institute, Praha.

Ledvinka, O. (2014b). Is the Cox-Stuart test for trend really insensitive to autocorrelation? In van Lanen, H. A. J., Demuth, S., and van der Heijden, A., editors, *7th Global FRIEND-Water Conference Poster Proceedings - Hydrology in a Changing World: Environmental and Human Dimensions*, pages 68–69. Wageningen University/UNESCO, Wageningen/Paris.

Lettenmaier, D. P. (1976). Detection of trends in water quality data from records with dependent observations. *Water Resources Research*, 12(5):1037–1046.

Libiseller, C. and Grimvall, A. (2002). Performance of partial Mann-Kendall tests for trend detection in the presence of covariates. *Environmetrics*, 13(1):71–84.

Luo, X. (2013). *GPS Stochastic Modelling: Signal Quality Measures and ARMA Process*. Springer Theses. Springer-Verlag, Berlin; Heidelberg.

Ly, S., Charles, C., and Degré, A. (2013). Different methods for spatial interpolation of rainfall data for operational hydrology and hydrological modeling at watershed scale: a review. *Biotechnologie, Agronomie, Société et Environnement*, 17(2):392–406.

Machiwal, D. and Jha, M. K. (2012). *Hydrologic Time Series Analysis: Theory and Practice*. Springer Netherlands, Dordrecht.

MacKinnon, J. G. (1996). Numerical distribution functions for unit root and cointegration tests. *Journal of Applied Econometrics*, 11(6):601–618.

Maidment, D. R., editor (1993). *Handbook of Hydrology*. McGraw-Hill, New York.

Majerčáková, O., Fendeková, M., and Lešková, D. (1997). The variability of hydrological series due to extreme climate conditions and the possible change of the hydrological characteristics with respect to potential climate change. In Gustard, A., Blazkova, S., Brilly, M., Demuth, S., Dixon, J., and van Lanen, H., editors, *Postojna Conference, 1997, FRIEND'97 - Regional Hydrology: Concepts and Models for Sustainable Water Resource Management*, IAHS Red Book 246, pages 59–66. IAHS Press, Wallingford, Oxfordshire, UK.

Makarava, N. (2012). *Bayesian Estimation of Self-Similarity Exponent*. Dissertation thesis, University of Potsdam, Potsdam.

Mandelbrot, B. B. and Wallis, J. R. (1968). Noah, Joseph, and operational hydrology. *Water Resources Research*, 4(5):909–918.

Mandelbrot, B. B. and Wallis, J. R. (1969a). Computer experiments with fractional Gaussian noises: part 1, averages and variances. *Water Resources Research*, 5(1):228–241.

Mandelbrot, B. B. and Wallis, J. R. (1969b). Computer experiments with fractional Gaussian noises: part 2, rescaled ranges and spectra. *Water Resources Research*, 5(1):242–259.

Mandelbrot, B. B. and Wallis, J. R. (1969c). Computer experiments with fractional Gaussian noises: part 3, mathematical appendix. *Water Resources Research*, 5(1):260–267.

Mandelbrot, B. B. and Wallis, J. R. (1969d). Robustness of the rescaled range R/S in the measurement of noncyclic long run statistical dependence. *Water Resources Research*, 5(5):967–988.

Mandelbrot, B. B. and Wallis, J. R. (1969e). Some long-run properties of geophysical records. *Water Resources Research*, 5(2):321–340.

Matalas, N. C. and Langbein, W. B. (1962). Information content of the mean. *Journal of Geophysical Research*, 67(9):3441–3448.

Matalas, N. C. and Sankarasubramanian, A. (2003). Effect of persistence on trend detection via regression. *Water Resources Research*, 39(12):1342.

McCuen, R. H. (2003). *Modeling Hydrologic Change: Statistical Methods*. Lewis Publishers, Boca Raton.

McLeod, A. (2011). Kendall: Kendall rank correlation and Mann-Kendall trend test.

McLeod, A. I. and Hipel, K. W. (1978). Preservation of the rescaled adjusted range - 1. a reassessment of the Hurst phenomenon. *Water Resources Research*, 14(3):491–508.

McLeod, A. I., Yu, H., and Krougly, Z. L. (2007). Algorithms for linear time series analysis: with R package. *Journal of Statistical Software*, 23(5):1–26.

McLeod, A. I., Yu, H., and Mahdi, E. (2012). Time Series Analysis with R. In *Time Series Analysis: Methods and Applications*, volume 30 of *Handbook of Statistics*, pages 661–712. Elsevier, Oxford, UK.

Milly, P. C. D., Betancourt, J., Falkenmark, M., Hirsch, R. M., Kundzewicz, Z. W., Lettenmaier, D. P., and Stouffer, R. J. (2008). Stationarity is dead: whither water management? *Science*, 319(5863):573–574.

Modarres, R. and Ouarda, T. B. M. J. (2013). Modeling rainfall–runoff relationship using multivariate GARCH model. *Journal of Hydrology*, 499:1–18.

Montanari, A. (2003). Long-range dependence in hydrology. In Doukhan, P., Oppenheim, G., and Taqqu, M. S., editors, *Theory and Applications of Long-range Dependence*, pages 461–472. Birkhäuser, Boston, Massachusetts.

Montanari, A., Rosso, R., and Taqqu, M. S. (1997). Fractionally differenced ARIMA models applied to hydrologic time series: identification, estimation, and simulation. *Water Resources Research*, 33(5):1035–1044.

Monteiro, A., Carvalho, A., Ribeiro, I., Scotto, M., Barbosa, S., Alonso, A., Baldasano, J. M., Pay, M. T., Miranda, A. I., and Borrego, C. (2012). Trends in ozone concentrations in the Iberian Peninsula by quantile regression and clustering. *Atmospheric Environment*, 56:184–193.

Mudelsee, M. (2007). Long memory of rivers from spatial aggregation. *Water Resources Research*, 43(1):W01202.

Nacházel, K., Starý, M., and Zezulák, J. (2004). *Use of the methods of artificial intelligence in water management*. Academia, Praha.

Nash, J. E. and Sutcliffe, J. V. (1970). River flow forecasting through conceptual models part I — a discussion of principles. *Journal of Hydrology*, 10(3):282–290.

Navarro, X., Porée, F., Beuchée, A., and Carrault, G. (2013). Performance analysis of Hurst exponent estimators using surrogate-data and fractional lognormal noise models: application to breathing signals from preterm infants. *Digital Signal Processing*, 23(5):1610–1619.

Önöz, B. and Bayazit, M. (2012). Block bootstrap for Mann-Kendall trend test of serially dependent data. *Hydrological Processes*, 26(23):3552–3560.

Partal, T. (2010). Wavelet transform-based analysis of periodicities and trends of Sakarya basin (Turkey) streamflow data. *River Research and Applications*, pages 695–711.

Partal, T. (2012). Wavelet analysis and multi-scale characteristics of the runoff and precipitation series of the Aegean region (Turkey). *International Journal of Climatology*, 32(1):108–120.

Partal, T. and Küçük, M. (2006). Long-term trend analysis using discrete wavelet components of annual precipitations measurements in Marmara region (Turkey). *Physics and Chemistry of the Earth, Parts A/B/C*, 31(18):1189–1200.

Patton, A., Politis, D. N., and White, H. (2009). Correction to "Automatic block-length selection for the dependent bootstrap" by D. Politis and H. White. *Econometric Reviews*, 28(4):372–375.

Percival, D. B. and Walden, A. T. (2000). *Wavelet Methods for Time Series Analysis*. Cambridge Series in Statistical and Probabilistic Mathematics. Cambridge University Press, Cambridge; New York.

Petrásek, J. (2008). *Bootstrap methods for dependent observations*. Diploma thesis, Faculty of Mathematics and Physics, Charles University in Prague, Praha.

Pettitt, A. N. (1979). A non-parametric approach to the change-point problem. *Applied Statistics*, 28(2):126.

Pfaff, B. (2008). *Analysis of Integrated and Cointegrated Time Series with R*. Use R! Springer, New York, NY, 2nd edition.

Phillips, P. C. B. and Perron, P. (1988). Testing for a unit root in time series regression. *Biometrika*, 75(2):335–346.

Politis, D. N. and White, H. (2004). Automatic block-length selection for the dependent bootstrap. *Econometric Reviews*, 23(1):53–70.

Rivard, C. and Vigneault, H. (2009). Trend detection in hydrological series: when series are negatively correlated. *Hydrological Processes*, 23(19):2737–2743.

Rivard, C., Vigneault, H., Piggott, A. R., Larocque, M., and Anctil, F. (2009). Groundwater recharge trends in Canada. *Canadian Journal of Earth Sciences*, 46(11):841–854.

Rutkowska, A. and Ptak, M. (2012). On certain stationarity test of hydrologic series. *Studia Geotechnica et Mechanica*, 34(1):51–63.

Salas, J. D. (1993). Analysis and modeling of hydrologic time series. In Maidment, D. R., editor, *Handbook of Hydrology*, pages 19.1–19.72. McGraw-Hill, New York.

Salas, J. D., Delleur, D. W., and Yevjevich, V. (1980). *Applied Modeling of Hydrologic Time Series*. Water Resources Publications, Littleton, Colorado.

Šalek, M. (2000). The radar and raingauge merge precipitation estimate of daily rainfall — First results in the Czech Republic. *Physics and Chemistry of the Earth, Part B: Hydrology, Oceans and Atmosphere*, 25(10-12):977–979.

Sang, Y.-F., Wang, Z., and Liu, C. (2013). Discrete wavelet-based trend identification in hydrologic time series. *Hydrological Processes*, 27(14):2021–2031.

Sang, Y.-F., Wang, Z., and Liu, C. (2014). Comparison of the MK test and EMD method for trend identification in hydrological time series. *Journal of Hydrology*, 510:293–298.

Santander Meteorology Group (2012). fume: FUME package.

Sczepanek, R. (2003). *Spatio-Temporal Structure of Precipitation in the Mountain Catchment*. Phd thesis, Cracow University of Technology, Kraków.

Sen, A. K. and Niedzielski, T. (2010). Statistical characteristics of riverflow variability in the Odra River basin, southwestern Poland. *Polish Journal of Environmental Studies*, 19(2):387–396.

Sen, P. K. (1968). Estimates of the regression coefficient based on Kendall's tau. *Journal of the American Statistical Association*, 63(324):1379–1389.

Šercl, P. (2008). Assessment of methods for area precipitation estimates. *Meteorological Bulletin*, 61(2):33–43.

Serinaldi, F. and Kilsby, C. G. (2015). Stationarity is undead: uncertainty dominates the distribution of extremes. *Advances in Water Resources*, 77:17–36.

Shahin, M., van Oorschot, H. J. L., and de Lange, S. J. (1993). *Statistical Analysis in Water Resources Engineering*. A.A. Balkema, Rotterdam; Brookfield VT.

Sneyers, R. (1990). *On the Statistical Analysis of Series of Observations*. Number 143 in Technical Note. World Meteorological Organization, Geneva.

Sohoulande Djebou, D. C., Singh, V. P., and Frauenfeld, O. W. (2014). Analysis of watershed topography effects on summer precipitation variability in the southwestern United States. *Journal of Hydrology*, 511:838–849.

Solanas, A., Manolov, R., and Sierra, V. (2010). Lag-one autocorrelation in short series: estimation and hypotheses testing. *Psicologica*, 31(2):357–381.

Srivastav, R. K. and Simonovic, S. P. (2014). An analytical procedure for multi-site, multi-season streamflow generation using maximum entropy bootstrapping. *Environmental Modelling & Software*, 59:59–75.

Stahl, K., Hisdal, H., Hannaford, J., Tallaksen, L. M., van Lanen, H. A. J., Sauquet, E., Demuth, S., Fendekova, M., and Jódar, J. (2010). Streamflow trends in Europe: evidence from a dataset of near-natural catchments. *Hydrology and Earth System Sciences*, 14(12):2367–2382.

Štěpánek, P. (2004). *Homogenization of Air Temperature Series in the Czech Republic During a Period of Instrumental Measurements.* Czech Hydrometeorological Institute, Praha.

Štěpánek, P., Zahradníček, P., Brázdil, R., and Tolasz, R. (2012). *Methodology of Control and Homogenization of Time Series in Climatology.* Czech Hydrometeorological Institute, Praha.

Štěpánek, P., Zahradníček, P., and Farda, A. (2013). Experiences with data quality control and homogenization of daily records of various meteorological elements in the Czech Republic in the period 1961-2010. *Idojaras*, 117(1):123–141.

Štěpánek, P., Zahradníček, P., and Huth, R. (2011). Interpolation techniques used for data quality control and calculation of technical series: an example of a Central European daily time series. *Idojaras*, 115(1-2):87–98.

Střeštík, J., Rožnovský, J., Štěpánek, P., and Zahradníček, P. (2014). The change of annual and seasonal precipitation totals in the Czech Republic during 1961-2012. In Rožnovský, J., Litschmann, T., Středa, T., and Středová, H., editors, *Extrémy oběhu vody v krajině.* Czech Hydrometeorological Institute, Praha.

Szolgayová, E., Arlt, J., Blöschl, G., and Szolgay, J. (2014). Wavelet based deseasonalization for modelling and forecasting of daily discharge series considering long range dependence. *Journal of Hydrology and Hydromechanics*, 62(1):24–32.

Szolgayova, E., Laaha, G., Blöschl, G., and Bucher, C. (2014a). Factors influencing long range dependence in streamflow of European rivers. *Hydrological Processes*, 28(4):1573–1586.

Szolgayova, E., Parajka, J., Blöschl, G., and Bucher, C. (2014b). Long term variability of the Danube River flow and its relation to precipitation and air temperature. *Journal of Hydrology*, 519:871–880.

Tabari, H., Taye, M. T., and Willems, P. (2015). Statistical assessment of precipitation trends in the upper Blue Nile River basin. *Stochastic Environmental Research and Risk Assessment*, 29(7):1751–1761.

Tallaksen, L. M. and van Lanen, H. A. J., editors (2004). *Hydrological Drought: Processes and Estimation Methods for Streamflow and Groundwater*. Number 48 in Developments in Water Science. Elsevier, Amsterdam; Boston.

Taqqu, M. S., Teverovsky, V., and Willinger, W. (1995). Estimators for long-range dependence: an empirical study. *Fractals*, 3(04):785–798.

Teegavarapu, R. S. V. (2014a). Missing precipitation data estimation using optimal proximity metric-based imputation, nearest-neighbour classification and cluster-based interpolation methods. *Hydrological Sciences Journal*, 59(11):2009–2026.

Teegavarapu, R. S. V. (2014b). Statistical corrections of spatially interpolated missing precipitation data estimates. *Hydrological Processes*, 28(11):3789–3808.

Thiébaux, H. J. and Zwiers, F. W. (1984). The interpretation and estimation of effective sample size. *Journal of Climate and Applied Meteorology*, 23(5):800–811.

Tolasz, R., editor (2007). *Climate Atlas of Czechia*. Czech Hydrometeorological Institute, Praha.

Torrence, C. and Compo, G. P. (1998). A practical guide to wavelet analysis. *Bulletin of the American Meteorological Society*, 79(1):61–78.

Trapletti, A. and Hornik, K. (2016). tseries: Time Series Analysis and Computational Finance.

Tyralis, H. (2016). HKprocess: Hurst-Kolmogorov Process.

Tyralis, H. and Koutsoyiannis, D. (2011). Simultaneous estimation of the parameters of the Hurst–Kolmogorov stochastic process. *Stochastic Environmental Research and Risk Assessment*, 25(1):21–33.

v. Buttlar, J., Zscheischler, J., and Mahecha, M. D. (2014). An extended approach for spatiotemporal gapfilling: dealing with large and systematic gaps in geoscientific datasets. *Nonlinear Processes in Geophysics*, 21(1):203–215.

Vajskebr, V., Řičicová, P., and Vlnas, R. (2013). Impact of the climate change on water and snow balance in the "Jizerské" Mountains in the Czech Republic. *Bodenkultur*, 64(3-4):113–119.

van Huijgevoort, M. H. J., Hazenberg, P., van Lanen, H. A. J., and Uijlenhoet, R. (2012). A generic method for hydrological drought identification across different climate regions. *Hydrology and Earth System Sciences*, 16(8):2437–2451.

Ventura, V., Paciorek, C. J., and Risbey, J. S. (2004). Controlling the proportion of falsely rejected hypotheses when conducting multiple tests with climatological data. *Journal of Climate*, 17(22):4343–4356.

Vinod, H. D. (2006). Maximum entropy ensembles for time series inference in economics. *Journal of Asian Economics*, 17(6):955–978.

Vinod, H. D. and López-de Lacalle, J. (2009). Maximum entropy bootstrap for time series: the meboot R package. *Journal of Statistical Software*, 29(5):1–19.

Vlnas, R. (2015). Observed changes of hydrological balance components regarding the available water resources. *Water Management Technical and Economical Information Journal*, 57(4-5):27–32.

Vlnas, R. and Fiala, T. (2010). Spatial and temporal variability of hydrological drought in the Czech Republic. In *SGEM2010 Conference Proceedings*, volume 2, pages 59–66, Varna, Bulgaria. International Multidisciplinary Scientific Geoconference.

von Storch, H. and Zwiers, F. W. (2001). *Statistical Analysis in Climate Research*. Cambridge University Press, Cambridge, UK; New York.

Wagesho, N., Goel, N., and Jain, M. (2012). Investigation of non-stationarity in hydro-climatic variables at Rift Valley lakes basin of Ethiopia. *Journal of Hydrology*, 444-445:113–133.

Wang, W., Vrijling, J. K., van Gelder, P. H. A. J. M., and Ma, J. (2006). Testing for nonlinearity of streamflow processes at different timescales. *Journal of Hydrology*, 322(1-4):247–268.

Wang, X. L., Chen, H., Wu, Y., Feng, Y., and Pu, Q. (2010). New techniques for the detection and adjustment of shifts in daily precipitation data series. *Journal of Applied Meteorology and Climatology*, 49(12):2416–2436.

Wijngaard, J. B., Klein Tank, A. M. G., and Können, G. P. (2003). Homogeneity of 20th century European daily temperature and precipitation series. *International Journal of Climatology*, 23(6):679–692.

Wilks, D. S. (2006). On "field significance" and the false discovery rate. *Journal of Applied Meteorology and Climatology*, 45(9):1181–1189.

Wilks, D. S. (2011). *Statistical Methods in the Atmospheric Sciences*. Number 100 in International Geophysics Series. Academic Press, Oxford; Waltham, MA, 3rd edition.

WMO (2008). *Guide to Hydrological Practices. Hydrology – From Measurement to Hydrological Information*, volume 1. World Meteorological Organization, Geneva, Switzerland, 6th edition.

Xu, Y. (2016). hyfo: Hydrology and Climate Forecasting.

Yevjevich, V. (1972). *Stochastic Processes in Hydrology*. Water Resources Publications, Fort Collins, Colorado, USA.

Yue, S., Kundzewicz, Z. W., and Wang, L. (2012). Detection of changes. In *Changes in Flood Risk in Europe*, number 10 in IAHS Spetial Publication, pages 387–408. CRC Press, Balkema.

Yue, S., Pilon, P., and Cavadias, G. (2002a). Power of the Mann–Kendall and Spearman's rho tests for detecting monotonic trends in hydrological series. *Journal of Hydrology*, 259(1-4):254–271.

Yue, S., Pilon, P., and Phinney, B. (2003). Canadian streamflow trend detection: impacts of serial and cross-correlation. *Hydrological Sciences Journal*, 48(1):51–63.

Yue, S., Pilon, P., Phinney, B., and Cavadias, G. (2002b). The influence of autocorrelation on the ability to detect trend in hydrological series. *Hydrological Processes*, 16(9):1807–1829.

Yue, S. and Wang, C. (2002a). The influence of serial correlation on the Mann–Whitney test for detecting a shift in median. *Advances in Water Resources*, 25(3):325–333.

Yue, S. and Wang, C. (2004). The Mann-Kendall test modified by effective sample size to detect trend in serially correlated hydrological series. *Water Resources Management*, 18(3):201–218.

Yue, S. and Wang, C. Y. (2002b). Regional streamflow trend detection with consideration of both temporal and spatial correlation. *International Journal of Climatology*, 22(8):933–946.

Zeleňáková, M., Purcz, P., Hlavatá, H., and Blišťan, P. (2015a). Climate change in urban versus rural areas. *Procedia Engineering*, 119:1171–1180.

Zeleňáková, M., Purcz, P., Hlavatá, H., Gargar, I., and Portela, M. M. (2014a). Statistical trends of precipitation in chosen climatic station in Slovakia and Libya. *WSEAS Transactions on Environment and Development*, 10(1):298–305.

Zeleňáková, M., Purcz, P., and Oravcová, A. (2015b). Trends in water quality in Laborec river, Slovakia. *Procedia Engineering*, 119:1161–1170.

Zeleňáková, M., Purcz, P., Soľáková, T., and Demeterová, B. (2012). Analysis of trends of low flow in river stations in eastern Slovakia. *Acta Universitatis Agriculturae et Silviculturae Mendelianae Brunensis*, 60(5):265–274.

Zeleňáková, M., Purcz, P., Sol'áková, T., and Simonová, D. (2014b). Assessment of low flows occurrence in chosen river stations in Slovakia. *WSEAS Transactions on Environment and Development*, 10(1):417–422.

Zhang, Y., Yu, H., and McLeod, A. I. (2013). Developments in maximum likelihood unit root tests. *Communications in Statistics - Simulation and Computation*, 42(5):1088–1103.

List of Figures

List of Abbreviations

ACF ... autocorrelation function
AIC ... Akaike Information Criterion
ALRT ... adjusted likelihood ratio test
ARIMA ... integrated autoregressive-moving average process (model)
ARMA ... autoregressive-moving average process (model)
BHMLLESS–MK test ... Bayley–Hammersley–Matalas–Langbein–Lettenmaier equivalent sample size Mann–Kendall test
CC ... climate change
CET ... Central European Time
CHMI ... Czech Hydrometeorological Institute
CS test ... Cox–Stuart test for trend
DF test ... Dickey–Fuller unit root test
DWD ... Deutscher Wetterdienst (German meteorological service)
EMD ... empirical mode decomposition
FARIMA (or ARFIMA) ... factionally integrated autoregressive-moving average process (model)
FDR ... false discovery rate
FGN ... fractional Gaussian noise
GESS–MK test ... generalized equivalent sample size Mann–Kendall test
GIS ... geographical information system (sometimes science as well)
HKp ... Hurst–Kolmogorov process
IMWM-NRI ... Institute of Meteorology and Water Management - National Research Institute (Polish hydrometeorological service)
KPSS test ... Kwiatkowski–Phillips–Schmidt–Shin stationarity test
LTP ... long-term persistence
MEB ... maximum entropy bootstrap
MK test ... Mann–Kendall test
MLR ... multiple linear regression
OLS ... ordinary least squares
PDAC ... precipitation double aggregation curve
PMCC ... Pearson moment correlation coefficient(s)
PMW test ... Pettitt–Mann–Whitney test
POT ... peaks over threshold
PP test ... Phillips–Perron unit root test
RHBN ... Reference Hydrometric Basin Network (a dataset used in Canada for climate change studies)
RRS ... rescaled range statistic
SNHT ... standard normal homogeneity test

SROC .... Spearman rank order correlation
SSA .... singular spectrum analysis
STP .... short-term persistence
TFPW–MK test .... trend-free pre-whitening Mann–Kendall test
TLM .... threshold level method
TSA .... time series analysis
UTC .... Universal Time Coordinated
WLS .... weighted least squares
YW–MK test .... BHMLLESS–MK test with autocorrelation coefficient strictly estimated from detrended series (Yue–Wang test)

Attachments

The next pages contain just the papers referred throughout the accompanying chapters. The order respects their time of publication or, as regards the last two papers, the time of their expected publication. The papers were published (A1–A7), are accepted for publication (A8), or they are under review (A9). The *Acta Hydrologica Slovaca* (AHS) and *Meteorology Hydrology and Water Management* (MHWM) journals are not indexed/abstracted in ISI Web of Knowledge or Scopus but, thematically, the topics addressed there undoubtedly belong here. At least, they both are peer-reviewed journals. The papers are as follows

A1 Matoušková, M., Kliment, Z., Ledvinka, O., and Královec, V. (2011). Application of selected statistical tests to detect changes in the rainfall and runoff regime. *Bodenkultur*, 62(1–4):95–100.

A2 Kliment, Z., Matoušková, M., Ledvinka, O., and Královec, V. (2011). Trend analysis of rainfall-runoff regimes in selected headwater areas of the Czech Republic. *Journal of Hydrology and Hydromechanics*, 59(1):36–50. doi: 10.2478/v10098-011-0003-y

A3 Ledvinka, O. (2015). Scaling of low flows in Czechia – an initial assessment. *Proceedings of the International Association of Hydrological Sciences*, 366:188–189. doi: 10.5194/piahs-366-188-2015

A4 Ledvinka, O. (2015). Evolution of low flows in Czechia revisited. *Proceedings of the International Association of Hydrological Sciences*, 369:87–95. doi: 10.5194/piahs-369-87-2015

A5 Ledvinka O. and Lamacova, A. (2015). Detection of field significant long-term monotonic trends in spring yields. *Stochastic Environmental Research and Risk Assessment*, 29(5):1463–1484. doi: 10.1007/s00477-014-0969-1

A6 Ledvinka, O. (2015). Nonstationarities in technical precipitation series in Czechia. *Acta Hydrologica Slovaca*, 16(TC 1):199–207.

A7 Wdowikowski, M., Kaźmierczak, B., and Ledvinka, O. (2016). Maximum daily rainfall analysis at selected meteorological stations in the upper Lusatian Neisse River basin. *Meteorology Hydrology and Water Management*, 4(1):53–63.

A8 Ledvinka, O., Wdowikowski, M., and Kaźmierczak, B. (2016). Return levels of daily rainfall maxima in the Upper Lusatian Neisse River basin considering possible nonstationarity. *Proceedings of the International Association of Hydrological Sciences*, 375. (accepted for publication)

A9 Kaźmierczak, B., Ledvinka, O., and Wdowikowski, M. (2016). Variability of Lusatian Neisse River basin daily rainfall extremes as an effect of climate change. *European Countryside*. (under review)